NASA CONTRACTOR REPORT

NASA CR-144

NASA CR-144

N65 13959
(ACCESSION NUMBER)

(THRU)

161
(PAGES)

1
(CODE)

CR-144
(NASA CR OR TMX OR AD NUMBER)

32
(CATEGORY)

FACILITY FORM 602

GPO PRICE $

OTS PRICE(S) $ 4.00

Hard copy (HC)

Microfiche (MF) .75

SYNTHESIS OF AN AIRFOIL AT SUPERSONIC MACH NUMBER

by Lucien A. Schmit, Jr., and William A. Thornton

Prepared under Grant No. NsG-110-61 *by*
CASE INSTITUTE OF TECHNOLOGY
Cleveland, Ohio
for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1965

SYNTHESIS OF AN AIRFOIL AT SUPERSONIC MACH NUMBER

By Lucien A. Schmit, Jr., and William A. Thornton

Prepared under Grant No. NsG-110-61 by
CASE INSTITUTE OF TECHNOLOGY
Cleveland, Ohio

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Office of Technical Services, Department of Commerce, Washington, D.C. 20230 -- Price $4.00

ABSTRACT 13959

Synthesis has been defined as the rational directed evolution of a system configuration which, in terms of a defined criterion, efficiently performs a set of specified functional purposes. This document presents the application of the synthesis concept to a system with an aeroelastic technology. Specifically, the system is a hollow symmetric double wedge airfoil. There are two design variables; airfoil thickness and chord length. Behavior constraints are root angle of attack, tip deflection, flutter Mach number, and root stress. Side constraints on the design parameters are provided. The basic criterion function is the total energy required to drive the airfoil through a sequence of flight conditions. Trade off of weight versus energy is studied using airfoil weight as an additional behavior constraint. The gradient steep-descent alternate step synthesis method is used. Numerical results of three example syntheses and a trade-off study are included. The results indicate that the synthesis concept may be applied successfully to an aeroelastic system. The study should be extended to consider the system as a plate structure rather than assuming beam type structural action. Author

ACKNOWLEDGEMENTS

The authors express their appreciation to the following for their complete cooperation:

The National Aeronautics and Space Administration which sponsored this research under Grant No. NsG-110-61.

The Case Computing Center.

The Case Engineering Design Center.

SYMBOLS

a_∞	free stream speed of sound (ft/sec)
A	area of gross cross section (ft^2)
A_o	area of internal cross-section (ft^2)
AMAX q	maximum allowable root angle of attack in q^{th} flight condition (rad.)
C	airfoil chord length (ft)
$C_{h_{ij}}$	bending flexibility influence coefficients (ft/lb)
$C_{\alpha_{ij}}$	torsional flexibility influence coefficients (rad/ft.lb)
d	wing skin thickness (feet)
D	total drag (lbs)
D_f	friction drag (lbs)
DMAX q	maximum allowable leading edge tip deflection in q^{th} flight condition (ft)
E	Young's modulus (lbs/ft^2)
G	shear modulus (lbs/ft^2)
$h(y), h_i$	elastic axis deflection (ft)
i	$\sqrt{-1}$ (when not used as an index)
k	reduced frequency
$K_{h_{ij}}$	bending stiffness influence coefficients (lbs/ft)
$K_{\alpha_{ij}}$	torsional stiffness influence coefficients (ft-lbs/rad)
ℓ	airfoil semi-span (ft)

SYMBOLS (continued)

L_{h_i}	lift perturbation at i^{th} airfoil segment (lbs)
L_i	lift supplied by i^{th} airfoil segment (lbs)
L_p	required airfoil payload (lbs)
$L_1 L_2 L_3 L_4$	dimensionless lift coefficients for flutter analysis
M	Mach number
M_F	flutter Mach number
M_{t_i}	aerodynamic twisting amount at the i^{th} segment (ft-lbs).
M_{α_i}	moment perturbation at i^{th} airfoil segment (ft-lbs)
$M_1 M_1 M_2 M_2$	dimensionless moment coefficients for flutter analysis
N	number of airfoil segments
p_∞	free stream atmospheric pressure (lbs/ft^2)
p_L	aerodynamic pressure on lower airfoil surface (lbs/ft^2)
p_U	aerodynamic pressure on upper airfoil surface (lbs/ft^2)
r_α	dimensionless mass radius of gyration about elastic axis
Re	Reynold's number
S	number of flight conditions
SMAX q	maximum allowable root stress in the q^{th} flight condition (lbs/ft^2)
t	time (sec)

SYMBOLS (Continued)

t_q	time in q^{th} flight condition
T	airfoil thickness (ft)
T_∞	free stream temperature (°R)
U	flight speed (ft/sec)
$w(x,y)$	airfoil middle surface deflection (ft)
w_T	leading edge tip deflection (ft)
WGT	actual total airfoil weight (lbs)
WMAX	maximum allowable airfoil weight (lbs)
x	chordwise coordinate
x_α	dimensionless distance from elastic axis to center of gravity
y	spanwise coordinate
z	transverse coordinate
$\alpha(y)$, α_i	elastic twist angle (rad)
α_o	root angle of attack (rad)
Δ	synthesis move size control parameter
λ	flutter frequency parameter = $(\omega_\alpha/\omega)^2$
μ	dimensionless mass density ratio
μ_a	absolute viscosity (lbs-sec^2/ft^2)
ξ_i	participation coefficient for flutter analysis
ρ	airfoil material density (lbs-sec^2/ft^4)
ρ_∞	free stream air density (lbs-sec^2/ft^4)
ρ_s	solidity ratio

SYMBOLS (Continued)

σ_y	flexure stress (lbs/ft^2)
$\sigma_{1,2}$	principal stresses at root (lbs/ft^2)
τ	thickness ratio (T/C)
τ_f	frictional stress (lbs/ft^2)
τ_{sy}	torsional shear stress (lbs/ft^2)
$(\tau_{yz})_V$	transverse shear stress (lbs/ft^2)
ϕ	normalized composite behavior constraint
Φ	energy criterion function (ft-lbs)
ω	harmonic oscillation frequency (rad/sec)
ω_h	bending vibration frequency of wing in vacuum (rad/sec)
ω_α	torsional vibration frequency of wing in vacuum (rad/sec)
Ω	frequency ratio $(\omega_h/\omega_\alpha)^2$

TABLE OF CONTENTS

Chapter I

INTRODUCTION

The broad objective of the structural synthesis research program is to bring an increasingly meaningful class of structural design problems within the grasp of rational and directed optimization in terms of realistic criteria.

Previous studies, such as the truss[1] and waffle plate[2] synthesis work have dealt with structural systems where static structural analysis and elastic stability theory provided the governing technology employed to predict the behavior of any particular design.

These early structural synthesis studies led to the idea that it would be interesting to explore the potential of the synthesis concept for engineering systems governed by technologies such as dynamics, aeroelasticity, and thermoelasticity.

The development of a synthesis capability for an automated optimum design of a simple shock isolator is reported in Ref. 3. The results reported in Ref. 3 illustrate the feasibility of applying the synthesis concept to a problem in which the governing technology is dynamics.

This document reports on the application of the synthesis concept to an engineering system where the governing technology is aeroelasticity.

Within the aeroelastic technology itself, it is customary to make a clear distinction between static aeroelastic analyses and dynamic aeroelastic analyses. In static aeroelastic analyses the accelerations vanish and problems reduce to the determination of deformation and stress distributions under constant (with time) aerodynamic forces. Dynamic aeroelastic analyses, on the other hand, involve aerodynamic forces which are functions of time. These time dependent forces induce accelerations into the system, and so inertial forces must be considered as well as aerodynamic and elastic forces.

Collar[4] has classified problems in Aeroelasticity by means of a "triangle of forces" (Fig. 1). Each type of aero-elastic phenomon may be located on the diagram according to its relation to the three pertinent forces, inertial (I), aerodynamic (A), and elastic (E). Therefore, for example, because lift (L) and drag (D) depend only on aerodynamic and elastic forces, they are located on the aeroelastic triangle as shown in Fig. 1. Likewise, because flutter (F) and buffeting (B) depend on all three forces, they are depicted in the center of the triangle. Other realms of technology may also be shown on the triangle. Mechanical vibrations (V) and dynamic stability of aircraft (DS), while not being aeroelastic phenomena, are shown on the diagram.

The technology for the system to be studied in this report involves (L), (D), and (F) in Fig. 1. In particular, it is a uniform hollow symmetric double wedge airfoil of semi-span ℓ

chord C, depth T, and solidity ratio ρ_s. The solidity ratio is defined as the ratio of the net cross-sectional area to the cross-sectional area of the entire section. Consider that the airfoil is subjected to a set of continuously changing flight conditions in the course of a mission. Assume that the total mission may be idealized as the sum of S sub-missions, each of which is defined by a set of prescribed parameters. Each discrete set of prescribed parameters defining a sub-mission will henceforth be known as a flight condition, and the prescribed parameters of a flight condition are flight altitude (ALT), Mach Number (M), time in flight (t), and required payload (L_p).

The behavior of the system under a series of flight conditions will be evaluated by the determination of certain pertinent quantities which hereafter will be termed behavior functions. In any one flight condition, these behavior functions will be assigned extreme values which may not be violated. They are, in effect, inequality constraints on the system. The behavior functions will be taken as root angle of attack (α_o), stress at root (σ), tip deflection at the leading edge (w_T), and flutter Mach number (M_F).

In order to distinguish between acceptable designs in an effort to determine the best design configuration, a criterion function (ϕ) must be introduced. An optimum value of this criterion function will define the best design. The basic criterion function in this study is the total energy required to complete a mission.

Both the behavior functions and the criterion function are complicated functions of the airfoil configuration variables in addition to depending on the flight conditions. Since this report considers a uniform airfoil of fixed semi-span, and solidity ratio, the airfoil configuration is given by chord length (C), and depth (T). These quantities will be called the design variables.

A succinct qualitative statement of the problem may now be given as: To determine the design variables (T,C) of a uniform, hollow, symmetric double wedge airfoil, subjected to a sequence of flight conditions, such that the behavior functions assume acceptable values and the criterion function is optimized.

The next three chapters will contain the analysis of the system. This analysis will permit prediction of the behavior of a proposed trial design. For a particular trial design, it must be possible to evaluate the following for each flight condition:

1. leading edge tip deflection (w_T)
2. root angle of attack (α_o)
3. root stress (σ)
4. flutter Mach number (M_F)
5. pressure drag (D_p)
6. friction drag (D_f)

Chapter II contains a discussion of the first three quantities above under the title "Static Aeroelastic Analysis". Chapter III contains a discussion of (4) above as "Dynamic Aeroelastic Analysis", and (5) and (6) above are steady state phemonena which will be discussed in Chapter IV under the title "Criterion Function".

The remaining three chapters are devoted to discussion of the synthesis technique used, results obtained, and conclusions, respectively.

The appendix contains an explanation of the computer program and a sample program listing.

Chapter II

STATIC AEROELASTIC ANALYSIS

2.1 Force and Displacement Distributions

Due to motion of the airfoil through the atmosphere, a pressure will be imposed on its upper and lower surfaces. The vertical component of the difference between the pressures on the upper and lower surfaces integrated over the airfoil surface area is defined as the lift of the airfoil.

Various aerodynamic theories give expressions for the determination of this pressure differential. The simplest of these to apply is the so-called "Piston Theory" of Ashley and Zartarian(5), whereby the pressure differential

$$\Delta p = p_L - p_U \tag{2.1}$$

at a point on the airfoil surface is dependent only on the deflection and velocity at that point. This theory is valid in the Mach number range

$$M \geq 2.5$$

which will constitute the range of interest in this study.

According to Mills(6), the pressure differential per unit area of the middle surface of the airfoil using Piston Theory may be taken as (see Fig. 3).

$$\Delta p = p_L - p_U = 2\gamma\, p_\infty M \left(\alpha_o - \frac{\partial w}{\partial x}\right) \left[1 + \frac{\gamma + 1}{2} M \frac{\partial Z_\delta}{\partial x}\right]$$

$$- 2 \frac{\gamma\, p_\infty}{a_\infty} \frac{\partial w}{\partial t} \left[1 + \frac{\gamma + 1}{2} M \frac{\partial Z_\delta}{\partial x}\right] \qquad (2.2)$$

where

p_∞ = free stream atmospheric pressure (lbs/ft^2)

γ = specific heat ratio $\simeq$ 1.4

α_o = root angle of attack (radians)

$w(x,y)$ = deflection of airfoil middle surface (ft.)

M = Mach number

$2Z_\delta(x)$ = airfoil thickness distribution (Z_δ is the equation of the surface of the airfoil)

a_∞ = free stream speed of sound (ft/sec)

Ma_∞ = U; U = airfoil velocity (ft/sec)

Since, at present, interest is to be focused on static aeroelastic phenomena, let

$$\frac{\partial w}{\partial t} = 0$$

Then (2.2) becomes

$$\Delta p = 2\gamma\ p_\infty M \left(\alpha_o - \frac{\partial w}{\partial x}\right) \left[1 + \frac{\gamma + 1}{2} M \frac{\partial Z_\delta}{\partial x}\right] \qquad (2.3)$$

The lift developed by the airfoil is found by integrating equation (2.3) over the area of the middle surface so that, for small angle of attack

$$\text{Lift} \equiv L = \int_A \Delta p(x,y)\, dx\, dy \qquad (2.4)$$

Assuming that the wing behaves primarily like a beam rather than a plate, the deformation of the middle surface may be represented as (see Fig. 3),

$$w(x,y) = h(y) - x\,\alpha(y) \qquad (2.5)$$

assuming there is no chordwise bending, and small twist angle $\alpha(y)$.

Now to perform the indicated integration in equation (2.4), introduce

$$Z_\delta = -\frac{T}{C}x + \frac{T}{2} \qquad 0 \leq x \leq \frac{C}{2} \qquad (2.6)$$

$$Z_\delta = \frac{T}{C}x + \frac{T}{2} \qquad -\frac{C}{2} \leq x \leq 0$$

Note that

$$\frac{\partial w}{\partial x} = -\alpha(y) \qquad (2.7)$$

From an examination of equations (2.3) and (2.7) it is apparent that lift L will not be dependent on the vertical deflection of the airfoil, but only on its angular rotation.

Performing the integration indicated in equation (2.4) over the chordwise variable x, the lift supplied is

$$L(y) = 2\gamma\, p_\infty\, MC\, [\alpha_o + \alpha(y)] \qquad (2.8)$$

per unit span. Assuming further that L(y) is approximately constant for some finite length of wing $\frac{\ell}{N}$, where N is the number of segments into which the continuous airfoil is discretized, equation (2.8) takes the form

$$L_i = 2\gamma \; p_\infty M (\alpha_o + \alpha_i) C \frac{\ell}{N} \tag{2.9}$$

The total lift supplied must equal the prescribed payload (L_p) plus the airfoil weight (WGT), as

$$(L_p + WGT) = L_T = \sum_{i=1}^{N} L_i \tag{2.10}$$

To determine L_i by equation (2.9), α_i, the elastic twist angle, must be known. Consider the aerodynamic moment per unit span as given by (Fig. 3c)

$$- M_t (y) = L(y) \, e(y) \tag{2.11}$$

or in terms of $\Delta p(x,y)$

$$M_t(y) = \int_A \Delta p(x,y) \, x \, dx \, dy \tag{2.12}$$

where the postive moment vector acts in the negative y direction. Performing the integration in (2.12) over x, as before, and assuming $\Delta p(x,y)$ approximately constant over the span interval $\frac{\ell}{N}$,

$$M_{t_i} = -\frac{1}{2} \gamma \, p_\infty M^2 \, TC \frac{\ell}{N} \left(\frac{\gamma + 1}{2}\right) (\alpha_o + \alpha_i) \tag{2.13}$$

Rewriting (2.9) and (2.13) in matrix form as

$$\vec{L} = k' \vec{\alpha}_o + k' \vec{\alpha} \tag{2.14}$$

$$\vec{M}_t = k \vec{\alpha}_o + k \vec{\alpha} \tag{2.15}$$

where

$$\vec{\alpha}_o = \begin{Bmatrix} \alpha_o \\ \circ \\ \circ \\ \circ \end{Bmatrix} \qquad \vec{\alpha} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \circ \\ \circ \\ \alpha_N \end{Bmatrix}$$

and the definitions of the scalar constants k' and k are obtained by comparison of (2.9) with (2.14) and (2.13) with (2.15).

Introducing torsional flexibility influence coefficients, $C_{\alpha_{ij}}$

$$\alpha_i = \sum_{i=1}^{N} C_{\alpha_{ij}} M_{t_j} \tag{2.16}$$

where the $C_{\alpha_{ij}}$ are given by[7,8]

$$C_{\alpha_{ij}} = \frac{2\sqrt{\tau^2 + 1}}{G\,T^2\,C\,d} \frac{\ell}{N} (i - \tfrac{1}{2}); \quad i = 1, ---, N \quad j \geq i \tag{2.17}$$

$$C_{\alpha_{ij}} = C_{\alpha_{ji}}$$

and where d is the skin thickness of the airfoil as given by

$$d = \frac{1}{2} C \left(\frac{\sqrt{\tau^2+1} - [(\tau^2+1) - \tau(\rho_s)\sqrt{\tau^2+1/\tau^2+2}]^{1/2}}{\sqrt{\tau^2+1/\tau^2+2}} \right) \quad (2.18)$$

and ρ_s is the prescribed solidity ratio. The solidity ratio is defined as (see Fig. 3a) the ratio of the area of the net cross section to the area of the entire cross section,

$$\rho_s = \frac{A-A_o}{A} = 1 - \frac{(T - 2d\sqrt{\tau^2+1})(C - 2d\sqrt{1+1/\tau^2})}{TC} \quad (2.19)$$

Also, using bending flexibility influence coefficients, $C_{h_{ij}}$,

$$h_i = \sum_{j=1}^{N} C_{h_{ij}} L_j \quad (2.20)$$

where

$$C_{h_{ij}} = \frac{8}{E}\left(\frac{\ell}{N}\right)^3 \left(\frac{1}{T^3C - (T-2d\sqrt{\tau^2+1})^3(C-2d\sqrt{1+1/\tau^2})} \right) [3(j-\tfrac{1}{2})(i-\tfrac{1}{2})^2$$

$$- (i-\tfrac{1}{2})^3 + \langle i-j \rangle^3]$$

$$C_{h_{ij}} = C_{h_{ji}} \qquad i = j = 1, ---, N \quad (2.21)$$

The last term in (2.21) is defined as

$$\langle i-j \rangle^3 \equiv (i-j)^3 \qquad i \geq j \quad (2.22)$$

$$< i - j >^3 \equiv 0 \qquad j > i \qquad (2.22\ \text{cont.})$$

An iterative technique must be resorted to in order to determine $\vec{\alpha}$, $\vec{M}_t$, $\vec{L}$, and $\vec{h}$. Casting (2.16) and (2.20) in matrix form gives

$$\vec{\alpha} = [C_\alpha] \vec{M}_t \qquad (2.23)$$

$$\vec{h} = [C_h] \vec{L} \qquad (2.24)$$

Then, from (2.15) and (2.23)

$$\vec{\alpha} = [C_\alpha] [k \vec{\alpha}_o + k \vec{\alpha}] \qquad (2.25)$$

As a first approximation for $\vec{\alpha}_o$, assume the wing is rigid, and thus that,

$$\Delta p = 2\gamma \; p_\infty M \left[1 + \frac{\gamma + 1}{2} M \frac{\partial Z_\delta}{\partial x}\right] \alpha_o \qquad (2.26)$$

$$L_T = \int \Delta p \, dA = 2\gamma \; \ell C \, p_\infty \, M \, \alpha_o$$

and since L_T is known when the design variables are known

$$\alpha_o^{(1)} = \frac{L_T}{2\gamma \; \ell C M \, p_\infty} \qquad (2.27)$$

The super (1) indicates that this is in fact only a first approximation to the actual value of α_o. Now from (2.25)

$$\vec{\alpha} = k \, [I - k \, [C_\alpha]]^{-1} \; [C_\alpha] \; \vec{\alpha}_o \qquad (2.28)$$

Substitution of (2.27) into (2.28) gives a first approximation to $\vec{\alpha}$, as

$$\vec{\alpha}^{(1)} = k\,[I - k\,[C_\alpha]]^{-1}\,[C_\alpha]\,\vec{\alpha}_o^{(1)} \tag{2.29}$$

and from (2.14)

$$\vec{L}^{(1)} = k'\,\vec{\alpha}_o^{(1)} + k'\,\vec{\alpha}^{(1)} \tag{2.30}$$

summing

$$L_T^{(1)} = \sum_{i=1}^{N} L_i^{(1)} \tag{2.31}$$

Since $L_T^{(1)}$ will not in general be equal to L_T until the correct twist distribution is obtained, the difference between $L_T^{(1)}$ and L_T is a measure of the correction needed. Then

$$\Delta L^{(1)} = L_T^{(1)} - L_T \tag{2.32}$$

and the correction to $\vec{\alpha}_o^{(1)}$ may be computed as

$$\Delta \alpha_o^{(1)} = \frac{\Delta L^{(1)}}{2\,\gamma\,\ell\,p_\infty\,CM} \tag{2.33}$$

and

$$\vec{\alpha}_o^{(2)} = \vec{\alpha}_o^{(1)} - \Delta\,\vec{\alpha}_o^{(1)} \tag{2.34}$$

The sequence of operations indicated by equations (2.29) through (2.34) is now repeated until

$$\left|\sum_{i=1}^{N} L_i^{(n)} - L_T\right| \leq \varepsilon \tag{2.35}$$

where ε is an arbitrary small value and n is the number of cycles required to satisfy equation (2.35). When (2.35) is satisfied, $\vec{\alpha}^{(n)}$ is the converged twist distribution and $\vec{L}^{(n)}$ is the converged lift distribution. Also the elastic axis deflection distribution, $\vec{h}^{(n)}$, may be computed from (2.24) and the aerodynamic moment $\vec{M}_t^{(n)}$ from (2.15). The deflection of the leading edge may be computed from (2.5) with $x = -\frac{C}{2}$, to give

$$\vec{w}_T^{(n)} = \vec{h}^{(n)} + \left(\frac{C}{2}\right) \vec{\alpha}^{(n)} \tag{2.36}$$

2.2 Stress Analysis

In part 2.1 of this chapter, the force and displacement distributions over the wing were determined. Now these will be utilized to determine the root stress.

The stresses at the root are

1) flexure stress due to L_i
2) transverse shear stress due to L_i
3) torsional shear stress due to M_{t_i}

1) Flexure Stress

This is given by

$$(\sigma_y)_{max} = \frac{M_r T}{2 I_{xx}} \tag{2.37}$$

where M_r = bending moment at root

I_{xx} = moment of inertia about the x axis at root.

Now

$$M_r = L_1 \frac{1}{2} \frac{\ell}{N} + L_2 \frac{3}{2} \frac{\ell}{N} + \cdots + L_N \left(N - \frac{1}{2}\right) \frac{\ell}{N}$$

Therefore

$$M_r = \frac{\ell}{N} \sum_{i=1}^{N} \left(i - \frac{1}{2}\right) L_i^{(n)} \tag{2.38}$$

and

$$I_{xx} = \int_A z^2 \, dA = \frac{1}{48} \left[CT^3 - \left(C - 2d\sqrt{1+1/\tau^2}\right)\left(T - 2d\sqrt{\tau^2+1}\right)^3\right] \tag{2.39}$$

so that finally,

$$(\sigma_y)_{max} = 24 \left(\frac{\ell}{N}\right) \left(\frac{T}{CT^3 - (C-2d\sqrt{1+1/\tau^2})(T-2d\sqrt{\tau^2+1})^3}\right) \sum_{i=1}^{N} \left(i - \frac{1}{2}\right) L_i^{(n)} \tag{2.40}$$

2) Shear Stress due to Transverse Load is

$$(\tau_{yz})_V = \frac{V\,Q}{2\, I_{xx}\, d\sqrt{1+1/\tau^2}} \tag{2.41}$$

The maximum value of (2.41) occurs at $z = 0$, and

$$Q = \int_{A_1} Z \, dA \tag{2.42}$$

where A_1 is the cross sectional area above the $z = 0$ plane. Then

$$Q = \frac{1}{12} TC\left(\frac{\rho_s}{2-\rho_s}\right) \left[T(2-\rho_s) - (1-\rho_s)\ d\sqrt{\tau^2+1}\ \right] \tag{2.43}$$

Also, $V = \sum_{i=1}^{N} L_i^{(n)}$, the converged total lift on the airfoil.

The stress is then

$$(\tau_{yz})_{Vmax} = 2\left(\frac{TC}{d\sqrt{1+1/\tau^2}}\right)\left(\frac{\rho_s}{2-\rho_s}\right)\left[\frac{T\ (2-\rho_s)-(1-\rho_s)d\ \sqrt{\tau^2+1}}{CT^3-(C-2d\sqrt{1+1/\tau^2})(T-2d\sqrt{\tau^2+1})^3}\right]\sum_{i=1}^{N} L_i^{(n)} \tag{2.44}$$

3) Shear Stress due to Torsion.

The torsional shear stress in the thin-walled, hollow wing due to a twisting moment M_t is given by[9]

$$\tau_{sy} = \frac{M_t}{2\ \bar{A}d} \tag{2.45}$$

where $\bar{A}$ is the cross sectional area taken to the center of the thin wall, as

$$\bar{A} = \frac{1}{2}\,(T - d\ \sqrt{\tau^2+1})\ \ (C - d\ \sqrt{1+1/\tau^2}) \tag{2.46}$$

and s is a circumferential coordinate in the plane of the cross section. Using (2.46), (2.45) becomes, at the wing root

$$\tau_{sy} = \frac{1}{(T-d\sqrt{\tau^2+1})\ \ (C-d\ \sqrt{1+1/\tau^2})d}\ \sum_{i=1}^{N} M_{t_i} \tag{2.47}$$

A numerical example was undertaken to determine the relative sizes of τ_{sy}, $(\tau_{yz})_{Vmax}$, and (σ_y). Both of the shear stresses were found to be small compared to σ_y, the bending stress.

The magnitude of the shear stress given by (2.47) is the same for every point in the wall of the wing root cross-section. Therefore, the principal stress composed of σ_y and τ_{sy} at point $x = 0$, $z = \frac{T}{2}$ will be taken to describe the root stress condition.

The principal stresses at this point are given by

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{sy})^2\right]^{\frac{1}{2}} \tag{2.48}$$

where $\sigma_x = 0$

Therefore, substitution of (2.47) and (2.40) into (2.48) yields

$$\sigma_{1,2} = \frac{12\ (\ell/N)\ T}{CT^3 - (C-2d\sqrt{1+1/\tau^2})(T-2d\sqrt{\tau^2+1})^3} \sum_{i=1}^{N} (i-\tfrac{1}{2})\ L_i^{(n)}$$

$$\pm \left[\left(\frac{12\ (\ell/N)\ T}{CT^3 - (C-2d\sqrt{1+1/\tau^2})(T-2d\sqrt{\tau^2+1})^3} \sum_{i=1}^{N} (i-\tfrac{1}{2})\ L_i^{(n)}\right)^2\right.$$

$$\left. + \left(\frac{1}{(T-d\sqrt{\tau^2+1})(C-d\sqrt{1+1/\tau^2})d} \sum_{i=1}^{N} M_{t_i}\right)^2\right]^{\frac{1}{2}} \tag{2.49}$$

A satisfactory design based on the Von Mises Criterion will be one for which

$$SMAX \geq [\sigma_1^{\,2} + \sigma_2^{\,2} - \sigma_1\,\sigma_2]^{\frac{1}{2}} \tag{2.50}$$

where SMAX is the yield stress in uniaxial tension.

Chapter III

DYNAMIC AEROELASTIC ANALYSIS

Flutter, which is the topic of this chapter, can be defined as the dynamic instability of an elastic body in an airstream. The flutter condition is determined by consideration of a perturbation in deflection about the deflected static equilibrium position of the airfoil. Due to this perturbation in deflections, the aerodynamic lift and moment distributions are changed. Under certain conditions, the magnitude of the deformation perturbations may grow with time and cause the failure of the wing.[11]

Consider that the wing is moving through air at some Mach number M, and is suddenly disturbed, as by a gust. Then the subsequent motion will either be damped out, remain constant, or increase. As the speed is increased from zero to some value just less than a critical Mach number, M_F, the perturbation will damp out. At the critical speed condition ($M = M_F$), neutral stability exists, and for speeds greater than the critical speed, divergent oscillations may result, which may cause structural failure. The flutter (or critical) Mach number, M_F, is therefore defined as the lowest Mach number at which a given structure flying at given atmospheric conditions will exhibit sustained oscillations about the deflected static equilibrium position.

In most cases, an adequate evaluation of the flutter condition is obtained by considering an infinitesimal perturbation about the deformed equilibrium position,[12] since it is an undesirable situation to have small motions unstable even if larger ones are stable. It is then sufficient to analyze a vibration with exponential time dependence, since all other small motions can be built up there from by superposition. Hence, theoretical flutter analysis usually consists of assuming in advance that all dependent variables are proportional to $e^{i\omega t}$ (ω real), and then finding combinations of M and ω for which this actually occurs. This leads to a complex or double eigenvalue problem where there are two characteristic numbers which determine Mach number and frequency.

With the above considerations, the perturbation in the deflection of the middle surface of the airfoil is given as (simple harmonic motion)

$$w(x,y,t) = \bar{w}(x,y)\, e^{i\omega t} \qquad (3.1)$$

where $\bar{w}(x,y)$ is in general complex. As before, take

$$w(x,y,t) = h(y,t) - x\,\alpha(y,t) \qquad (3.2)$$

or

$$\bar{w}(x,y)\, e^{i\omega t} = \bar{h}(y)\, e^{i\omega t} - x\,\bar{\alpha}(y) e^{i\omega t} \qquad (3.3)$$

From piston theory,[6] the aerodynamic pressure is

$$\Delta p(x,y,t) = 2\gamma\, p_\infty M \frac{\partial w}{\partial x}\left[1 + \frac{\gamma + 1}{2} M \frac{\partial Z_\delta}{\partial x}\right]$$

$$- 2 \frac{p_\infty \gamma}{a_\infty}\left[1 + \left(\frac{\gamma + 1}{2}\right) M \frac{\partial Z_\delta}{\partial x}\right] \frac{\partial w}{\partial t} \tag{3.4}$$

Using the thickness distribution equations and definition of lift as given in Chapter II, the lift perturbation per wing segment is

$$L_{h_i}(t) = \omega^2 \left\{ \left[0 - i\left(\frac{\gamma\, p_\infty}{M\, a_\infty^2} \frac{C^2}{k} \frac{\ell}{N}\right)\right] \bar{h}_i\, e^{i\omega t} \right. \tag{3.5}$$

$$\left. + \left[\left(\frac{\gamma}{2} \frac{p_\infty}{a_\infty^2} \frac{C^3}{M} \frac{1}{k^2} \frac{\ell}{N}\right) - i\left(\frac{p_\infty}{4\, a_\infty^2} \gamma \frac{(\gamma + 1)}{2} T C^2 \frac{\ell}{N} \frac{1}{k}\right)\right] \bar{\alpha}_i\, e^{i\omega t} \right\}$$

where

$$k = \frac{(C/2)\, \omega}{M\, a_\infty} \tag{3.6}$$

and ω is the circular frequency of harmonic motion in radians per second. As is seen from Eq. (3.6) for reduced frequency k, ω and M are directly related.

In addition to the aerodynamic lift acting on the airfoil, there will also be a perturbation in the aerodynamic moment, because the resultant lift perturbation will not of necessity act through the elastic axis (mid-chord). This moment may be determined by evaluating the integral

$$M_{\alpha_i} = \frac{\ell}{N} \int_{-\frac{C}{2}}^{+\frac{C}{2}} \Delta p_i \; x \, dx$$

which gives, after some manipulation

$$M_{\alpha_i} = \omega^2 \Bigg\{ \left[0 + i \left(-\frac{\gamma}{4} \left(\frac{\gamma + 1}{2} \right) \frac{p_\infty}{a_\infty^{\;2}} \, TC^2 \, \frac{1}{k} \frac{\ell}{N} \right) \right] \bar{h}_i \, e^{i\omega t} \qquad (3.7)$$

$$+ \left[- \left(-\frac{\gamma}{8} \left(\frac{\gamma + 1}{2} \right) \frac{p_\infty}{a_\infty^{\;2}} \, \frac{TC^3}{k^2} \, \frac{\ell}{N} \right) + i \left(\frac{1}{12} \frac{\ell}{N} \gamma \, \frac{p_\infty}{a_\infty^{\;2} M} \, \frac{C^4}{k} \right) \right] \bar{\alpha}_i \, e^{i\omega t} \Bigg\}$$

Defining harmonic lift and moment quantities as

$$L_{h_i} = \bar{L}_{h_i} \, e^{i\omega t} \qquad (3.8)$$

$$M_{\alpha_i} = \bar{M}_{\alpha_i} \, e^{i\omega t} \qquad (3.9)$$

the factor $e^{i\omega t}$ in (3.5) and (3.7) may be dropped. Equations (3.5) and (3.7) are commonly written in current literature as

$$\bar{L}_{h_i} = \frac{1}{2} \rho_\infty C^3 \frac{\ell}{N} \omega^2 \left[-2 \left(L_1 + i \, L_2 \right) \frac{\bar{h}_i}{C} + \left(L_3 + i \, L_4 \right) \bar{\alpha}_i \right] \qquad (3.10)$$

$$\bar{M}_{\alpha_i} = \frac{1}{4} \rho_\infty C^4 \frac{\ell}{N} \omega^2 \left[-2 \left(M_1 + i \, M_2 \right) \frac{\bar{h}_i}{C} + \left(M_3 + i \, M_4 \right) \bar{\alpha}_i \right] \qquad (3.11)$$

The coefficients L_1, L_2, L_3, L_4 and M_1, M_2, M_3, M_4 are dimensionless lift and moment coefficients which are defined by comparison of (3.5) with (3.10), and (3.7) with (3.11).

The equations of motion of the basic airfoil segment of length $\frac{\ell}{N}$ will now be determined. The forces acting and the sign convention used are shown in Fig. 4. It is assumed that the center of gravity is not coincident with the elastic axis but is located by the parameter x_α (positive behind elastic axis). This is reasonable because in practice there will be a control surface actuation system near the trailing edge. It is assumed that this will not affect the airfoil structurally. Note that Bisplinghoff[12] states that no flutter will occur at supersonic speeds unless the center of gravity is behind the elastic axis.

The translational and torsional springs with constants indicated by K_h and K_α are meant to indicate the stiffness influence coefficients of the particular airfoil section being considered. Constructing the potential and kinetic energies,

$$V = \frac{1}{2} K_h h_i^2 + \frac{1}{2} K_\alpha \alpha_i^2 \tag{3.12}$$

$$T = \frac{1}{2} \int (\dot{w}_i)^2 \, dm = \frac{1}{2} \int (\dot{h}_i - x \, \dot{\alpha}_i)^2 \, dm$$

$$T = \frac{1}{2} (\dot{h}_i)^2 \int dm - \dot{h} \, \dot{\alpha}_i \int x \, dm + \frac{1}{2} (\dot{\alpha}_i) \int x^2 \, dm \tag{3.13}$$

Applying Lagrange's Equation

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} + \frac{\partial V}{\partial q_k} = F_k \tag{3.14}$$

for $q_k = h_i$, $F_k = L_{h_i}$

$$m \ddot{h}_i - S_\alpha \ddot{\alpha}_i + K_h h_i = L_{h_i} \tag{3.15}$$

and for $q_k = \alpha_i$, $F_k = - M_{\alpha_i}$

$$I_\alpha \ddot{\alpha}_i - S_\alpha \ddot{h}_i + K_\alpha \alpha_i = - M_{\alpha_i} \tag{3.16}$$

where

$$S_\alpha = \int x \, dm = m \left(\frac{C}{2}\right) x_\alpha \tag{3.17}$$

$$I_\alpha = \int x^2 dm = m \left(\frac{C}{2}\right)^2 r_\alpha^{\,2} \tag{3.18}$$

$$m = \int dm \tag{3.19}$$

m is the mass per $\frac{\ell}{N}$ length of airfoil

r_α is the dimensionless mass radius of gyration

x_α is the dimensionless static unbalance

Another way of deriving the differential equations of motion is by means of the flexibility influence coefficients defined earlier. Then the total deflection at section i may be written by superposition as

$$h_i = \sum_{j=1}^{N} C_{h_{ij}} F_j$$

where F_j is the total external force (aerodynamic and inertial) acting at section j. Therefore

$$h_i = C_{h_{ij}} (- m \ddot{h}_j + S_\alpha \ddot{\alpha}_j + L_{h_j}) \qquad (3.20)$$

Likewise, the total deflection angle at i is

$$\alpha_i = \sum_{j=1}^{N} C_{\alpha_{ij}} M_{T_j}$$

where M_{T_j} is the total moment at section j. Then

$$\alpha_i = C_{\alpha_{ij}} (- I_\alpha \ddot{\alpha}_j + S_\alpha \ddot{h}_j - M_{\alpha_j}) \qquad (3.21)$$

Comparison of equations (3.20) and (3.21) with (3.15) and (3.16) respectively indicates that K_h and K_α are matrices related by

$$[K_h] = [C_h]^{-1} \qquad (3.22)$$

$$[K_\alpha] = [C_\alpha]^{-1} \qquad (3.23)$$

and K_h and K_α are stiffness coefficient matrices as described above.

Now assuming that the motion of the airfoil may be represented as a simple harmonic motion, the motion equations become

$$K_{h_{ij}} \bar{h}_j - m \omega^2 \bar{h}_i + S_\alpha \omega^2 \bar{\alpha}_i - \bar{L}_{h_i} = 0 \qquad (3.24)$$

$$K_{\alpha_{ij}} \bar{\alpha}_j - I_\alpha \omega^2 \bar{\alpha}_i + S_\alpha \omega^2 \bar{h}_i + M_{\alpha_i} = 0 \qquad (3.25)$$

which may be written in matrix form as

$$\begin{bmatrix} K_h & 0 \\ 0 & K_\alpha \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} - \omega^2 \begin{bmatrix} m & 0 \\ 0 & I_\alpha \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} + \omega^2 \begin{bmatrix} 0 & S_\alpha \\ S_\alpha & 0 \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} + \begin{Bmatrix} -\bar{L}_{\alpha i} \\ \bar{M}_{\alpha i} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3.26}$$

By multiplying and dividing the first term in (3.26) by ω^2, a more convenient form is obtained as

$$\omega^2 \begin{bmatrix} [\frac{K_h}{\omega^2}] - [m] & [S_\alpha] \\ [S_\alpha] & [\frac{K_\alpha}{\omega^2}] - [I_\alpha] \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} + \begin{Bmatrix} -\bar{L}_h \\ \bar{M}_\alpha \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3.27}$$

Now replacing $\bar{L}_h$ and $\bar{M}_\alpha$ by their values as given by equations (3.10) and (3.11), and using (3.17) and (3.18)

$$\omega^2 \begin{bmatrix} [\frac{K_h}{\omega^2}] - [m] + [\rho_\infty C^2 \frac{\ell}{N} (L_1 + iL_2)] & [m \frac{C}{2} x_\alpha] - [\frac{1}{2}\rho_\infty C^3 \frac{\ell}{N}(L_3 + iL_4)] \\ [m \frac{C}{2} x_\alpha] - [\frac{1}{2}\rho_\infty C^3 \frac{\ell}{N}(M_1 + iM_2)] & [\frac{K_\alpha}{\omega^2}] - [m \frac{C^2}{4} r_\alpha^2] + [\frac{1}{4}\rho_\infty C^4 \frac{\ell}{N}(M_3 + iM_4)] \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

(3.28)

The matrix equation (3.28) is most easily solved if it is rendered dimensionless. In order to do this some new parameters must be defined. Dimensionless center of gravity location and the radius of gyration have been defined by

$$x_\alpha = \frac{2}{C} \frac{S_\alpha}{m} \tag{3.17}$$

$$r_\alpha = \left[\frac{4}{C^2} \frac{I_\alpha}{m} \right]^{1/2} \tag{3.18}$$

Next define a dimensionless vertical deflection given by

$$\bar{v} \equiv 2 \frac{\bar{h}}{C} \tag{3.29}$$

Then take dimensionless bending and torsion stiffness matrices, $[\tilde{K}_h]$ and $[\tilde{K}_\alpha]$, as

$$[K_h] = \omega_h^2 \; m \; [\tilde{K}_h] \tag{3.30}$$

$$[K_\alpha] = \omega_\alpha^2 \; m \; \frac{C^2}{4} \; r_\alpha^2 \; [\tilde{K}_\alpha] \tag{3.31}$$

where ω_h is the fundamental natural bending vibration frequency, and ω_α is the fundamental natural torsional vibration frequency of the wing in a vacuum. The basis for the definitions as given in Eq. (3.30) and (3.31) is the analogy drawn between the single degree of freedom systems where $\omega_h = \sqrt{K_h/m}$ and $\omega_\alpha = \sqrt{K_\alpha/I_\alpha}$ and the many degree of freedom system considered here. Finally, define a dimensionless mass density ratio as

$$\mu = \frac{m}{\rho_\infty C^2} \frac{N}{\ell} \qquad (3.32)$$

and the quantity m is of course given by

$$m = \frac{1}{2} \rho \frac{\ell}{N} TC \rho_s \qquad (3.33)$$

where ρ is the mass density of the material out of which the airfoil is constructed and ρ_s is the solidity ratio. In terms of densities, then,

$$\mu = \frac{1}{2} \frac{\rho}{\rho_\infty} \frac{T}{C} \rho_s \qquad (3.34)$$

Now by substitution of definitions (3.29) through (3.32) into equation (3.28), the dimensionless flutter equation becomes

$$\left[\begin{array}{c|c} \left(\frac{\omega_h}{\omega}\right)^2 \mu[\tilde{K}_h] - [\mu] + [L_1+iL_2] & [\mu\, x_\alpha] - [L_3+iL_4] \\ \hline [\mu\, x_\alpha] - [M_1+iM_2] & \mu r_\alpha^2 \left(\frac{\omega_\alpha}{\omega}\right)^2 [\tilde{K}_\alpha] - [\mu\, r_\alpha^2] + [M_3+iM_4] \end{array}\right] \left\{\begin{array}{c} \bar{v} \\ \hline \bar{\alpha} \end{array}\right\} = \left\{\begin{array}{c} 0 \\ \hline 0 \end{array}\right\}$$

(3.35)

The dimensionless lift and moment coefficients are given with $\tau = T/C$, the thickness ratio, as

$$\begin{aligned} L_1 &= 0 \\ L_2 &= \frac{1}{Mk} \\ L_3 &= \frac{1}{Mk^2} \end{aligned} \qquad (3.36)$$

$$L_4 = -\frac{6}{10}\frac{\tau}{k}$$

$$M_1 = 0$$

$$M_2 = -\frac{6}{10}\frac{\tau}{k} \qquad (3.36\text{ cont})$$

$$M_3 = -\frac{6}{10}\frac{\tau}{k^2}$$

$$M_4 = \frac{1}{3\,Mk}$$

Substitution of (3.36) into (3.35) gives the flutter equation in its final form

$$\left[\begin{array}{c|c} \mu[(\frac{\omega_h}{\omega_\alpha})^2(\frac{\omega_\alpha}{\omega})^2[\tilde{K}_h]-[I]] + [i\,\frac{1}{Mk}] & [\mu x_\alpha]-[\frac{1}{Mk^2}]+[i\,\frac{6}{10}\frac{\tau}{k}] \\ \hline [\mu x_\alpha]+[i\,\frac{6}{10}\frac{\tau}{k}] & \mu r_\alpha^{\,2}[(\frac{\omega_\alpha}{\omega})^2[\tilde{K}_\alpha]-[I]]-[\frac{6}{10}\frac{\tau}{k^2}] + [i\,\frac{1}{3Mk}] \end{array}\right]\left\{\begin{array}{c}\bar{v}\\ \hline \bar{\alpha}\end{array}\right\} = \left\{\begin{array}{c}0\\ \hline 0\end{array}\right\} \qquad (3.37)$$

The matrix equation (3.37) is the flutter equation which will be solved. Notice that most of the submatrices on the left are diagonal, there being only two exceptions, $[\tilde{K}_h]$ and $[\tilde{K}_\alpha]$. Notice also that (3.37) is a homogeneous equation in the displacements. If Eq. (3.37) is written in matrix form as

$$[A]\left\{\begin{array}{c}\bar{v}\\ \bar{\alpha}\end{array}\right\} = \left\{\begin{array}{c}0\\ 0\end{array}\right\} \qquad (3.37a)$$

or in vector form

$$A\vec{v} = \vec{0}; \qquad \vec{v} = \begin{Bmatrix} \bar{V} \\ \\ \bar{\alpha} \end{Bmatrix} \tag{3.37b}$$

then nontrivial solutions for (3.37) will exist if and only if

$$\det[A] \equiv 0 \tag{3.38}$$

The matrix A will be taken here to have 10 rows and 10 columns, corresponding to the five sections into which the airfoil will be discretized. This is sufficient freedom for the determination of the fundamental mode and flutter frequency. Because there will be both a bending and a torsional degree of freedom, the total number of equilibrium equations will be ten. Each of these ten equations has a real and an imaginary part. This effectively gives the equivalent of twenty equations for only five airfoil stations. This is a practical consideration for the choice of five airfoil stations being sufficient. The solution with five wing segments requires the expansion of a tenth order complex determinant. To complicate matters, neither ω nor k nor M are known until the problem is solved. The solution then is one of trial and error, where ω and M are guessed and k computed from Eq. (3.6), and then these quantities substituted into (3.38), the determinant is expanded, evaluated, and compared with zero. Obviously there is little chance that it will be zero, and because there are two quantities (ω and M) to be arbitrarily chosen, it will take many trials to get an answer. There is no way around this dilemma unless simplifying assumptions are made. One such assumption is that the displacement vector may be written as a sum of the uncoupled bending and torsional modes of the segmented airfoil, as

$$\vec{v} = \sum_{i=1}^{5} \bar{\xi}_{h_i} \vec{e}_{h_i} + \sum_{i=1}^{5} \bar{\xi}_{\alpha_i} \vec{e}_{\alpha_i} \tag{3.39}$$

where the $\bar{\xi}_i$ are normal coordinates which indicate the participation of the several normal modes of the system in the general solution vector. They are therefore also called participation coefficients. The $\vec{e}_i$ are the normal modes which are the solutions to the following eigenvalue problems:

$$\vec{e}_{h_i} : \quad \vec{h} = \omega_h^2 \, m \, [C_h] \, \vec{h} \tag{3.40}$$

Therefore

$$\vec{e}_{h_i} = \omega_{h_i}^2 \, m \, [C_h] \, \vec{e}_{h_i}$$

$$\vec{e}_{\alpha_i} : \quad \vec{\alpha} = \omega_\alpha^2 \, I_\alpha \, [C_\alpha] \, \vec{\alpha} \tag{3.41}$$

Therefore:

$$\vec{e}_{\alpha_i} = \omega_{\alpha_i}^2 \, I_\alpha \, [C_\alpha] \, \vec{e}_{\alpha_i}$$

Eq. (3.39) may then be written in matrix form as

$$\underset{10x1}{\vec{v}} = \underset{10x10}{\left[\begin{array}{c|c} Q_h & 0 \\ \hline 0 & Q_\alpha \end{array}\right]} \underset{10x1}{\vec{\xi}} \equiv Q\,\vec{\xi}; \quad \vec{\xi} = \left\{\begin{array}{c} \vec{\xi}_h \\ -- \\ \vec{\xi}_\alpha \end{array}\right\} \tag{3.42}$$

So far nothing has been done to eleviate the dilemma involved in solving Eq. (3.38). However, since the problem requires only the fundamental flutter mode shape, it will be assumed not only that $\vec{v}$ may be written as in (3.42), but also that only the first few uncoupled bending and torsion modes will make up this fundamental flutter mode shape. In fact, only the fundamental uncoupled bending mode and the fundamental uncoupled torsion mode will be taken to approximate $\vec{v}$. Therefore

$$\underset{10\times1}{\vec{v}} = \underset{10\times2}{\left[\begin{array}{c|c} e_{h_1} & 0 \\ \hline 0 & e_{\alpha_1} \end{array}\right]} \underset{2\times1}{\vec{\xi}} \tag{3.43}$$

A separate analysis was undertaken which showed the justification of this. In fact, the results indicated that

$$\frac{\xi_{h_1}}{\xi_{h_2}} \cong \frac{\xi_{\alpha_1}}{\xi_{\alpha_2}} \cong \frac{100}{1} \tag{3.44}$$

so that the participation of the second modes is of higher order of magnitude than the first. Higher modes would be expected to have even smaller participation coefficients.

A way has now been shown to reduce the order of the determinant in Eq. (3.38). Substitution of Eq. (3.43) into (3.37b) yields

$$\underset{10\text{x}10}{[A]} \quad \underset{10\text{x}2}{[Q]} \quad \underset{2\text{x}1}{\vec{\xi}} \quad = \quad \underset{10\text{x}1}{\vec{0}} \tag{3.45}$$

and premultiplying both sides of (3.45) by Q^T gives

$$\underset{2 \times 2}{[Q^T \, A \, Q]} \quad \underset{2\text{x}1}{\vec{\xi}} \quad = \quad \underset{2\text{x}1}{\vec{0}} \tag{3.45a}$$

and then (3.38) may be replaced by

$$\det \, [Q^T \, A \, Q] \; = \; 0 \tag{3.46}$$

Now the determinant in (3.46) is only of second order and may be easily expanded.

Next the matrix multiplications indicated in equation (3.45a) are performed. Because of the form of matrix Q, that is because

$$Q = \left[\begin{array}{c|c} Q_h & 0 \\ \hline 0 & Q_\alpha \end{array}\right]; \qquad \begin{array}{l} \{Q_h\} = \vec{e}_{h_1} \\ \\ \{Q_\alpha\} = \vec{e}_{\alpha_1} \end{array}$$

the multiplications are especially simple. If [A] is partitioned as

$$[A] = \left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right] \tag{3.47}$$

then

$$\left[\begin{array}{c|c} Q_h^T & 0 \\ \hline 0 & Q_\alpha^T \end{array}\right] \left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right] \left[\begin{array}{c|c} Q_h & 0 \\ \hline 0 & Q_\alpha \end{array}\right] = \left[\begin{array}{c|c} Q_h^T A_1 Q_h & Q_h^T A_2 Q_\alpha \\ \hline Q_\alpha^T A_3 Q_h & Q_\alpha^T A_4 Q_\alpha \end{array}\right] \qquad (3.48)$$

and all of the submatrices on the right side of (3.48) are of first order and are therefore individual complex numbers.
Now, looking back at Eq.(3.37) , it is seen that every sub-matrix may be written as a scalar parameter multiplied by a unit matrix except $[\tilde{K}_h]$ and $[\tilde{K}_\alpha]$. Then, defining the following

$$\begin{aligned} Q_h^T \ [\tilde{K}_h] \ Q_h &\equiv \tilde{K}_h \\ Q_\alpha^T \ [\tilde{K}_\alpha] \ Q_\alpha &\equiv \tilde{K}_\alpha \\ Q_h^T \ [I] \ Q_h &\equiv Q_1 \\ Q_h^T \ [I] \ Q_\alpha &\equiv Q_2 \\ Q_\alpha^T \ [I] \ Q_h &\equiv Q_3 \\ Q_\alpha^T \ [I] \ Q_\alpha &\equiv Q_4 \end{aligned} \qquad (3.49)$$

and writing

$$\left(\frac{\omega_h}{\omega_\alpha}\right)^2 = \Omega \qquad (3.50)$$

$$\left(\frac{\omega_\alpha}{\omega}\right)^2 = \lambda \qquad (3.51)$$

equation (3.46) becomes

$$\begin{vmatrix} [\mu \quad (\tilde{K}_h\Omega\lambda - Q_1) + i\,\frac{Q_1}{Mk}] & [Q_2\ \mu x_\alpha - \frac{Q_2}{Mk^2} + i\,\frac{6}{10}\,\frac{\tau}{k}\ Q_2] \\ [Q_3\ \mu x_\alpha + i\,\frac{6}{10}\,\frac{\tau}{k}\,Q_3] & [\mu\ r_\alpha^{\ 2}(\tilde{K}_\alpha\lambda - Q_4) - \frac{6}{10}\ \frac{\tau}{k^2}\,Q_4 + i\,\frac{1}{3Mk}\,Q_4] \end{vmatrix} = 0 \tag{3.52}$$

When expanding (3.52), terms may be grouped according to whether they are real or imaginary. Then in order for the determinant to vanish, both the real and imaginary parts must vanish separately, thus giving two equations. Setting the real part equal to zero gives

$$\mu^2\ r_\alpha^{\ 2}\ (\tilde{K}_h\ \Omega\lambda - Q_1)(\tilde{K}_\alpha\lambda - Q_4) + \frac{1}{k^2}\,[-\,\frac{6}{10}\,\mu\ Q_4\tau(\tilde{K}_h\Omega\lambda\ - Q_1)$$

$$-\ \frac{Q_1\ Q_4}{3\ M^2} + Q_2Q_3\ \frac{\mu\ x_\alpha}{M} + \ \frac{18}{50}\,Q_2Q_3\tau^2\,] - Q_2Q_3\mu^2x_\alpha^2\ = \ 0 \tag{3.53}$$

and the imaginary part set equal to zero yields

$$\frac{\mu\ Q_4}{3}\ (\tilde{K}_h\Omega\lambda - Q_1) + \mu\ Q_1\ r_\alpha^{\ 2}\ (\tilde{K}_\alpha\lambda - Q_4)\ - \frac{6}{10}\,\frac{\tau}{k^2}\,(Q_1Q_4 - \ Q_2Q_3)$$

$$-\,\frac{6}{5}\ Q_2\ Q_3\mu\ x_\alpha\ \ M\tau\ \ = \ \ 0 \tag{3.54}$$

It will be noticed that k^2 appears only to the first power in both (3.53) and (3.54). Solving (3.54) for $1/k^2$, it is found that

$$\frac{1}{k^2} = \frac{5}{9}\frac{\mu Q_4}{\tau Q_o}(\tilde{K}_h\Omega\lambda - Q_1) + \frac{5}{3}Q_1\frac{\mu r_\alpha^2}{Q_o\tau}(\tilde{K}_\alpha\lambda - Q_4) - \frac{2Q_2Q_3}{Q_o}\mu x_\alpha M \tag{3.55}$$

where

$$Q_o = Q_1 Q_4 - Q_2 Q_3 \tag{3.56}$$

Substituting (3.55) into (3.53) and grouping terms as powers of λ, the following quadratic equation in λ is obtained;

$$\begin{aligned}
&\lambda^2 \left[\mu r_\alpha^2 \tilde{K}_h\tilde{K}_\alpha\Omega Q_o\tau - \frac{1}{3}\mu Q_4^2 \tilde{K}_h^2 \Omega^2\tau - Q_1Q_4\mu r_\alpha^2 \tilde{K}_h \tilde{K}_\alpha\Omega\tau\right] \\
&+ \lambda \Big[- Q_4Q_o\mu r_\alpha^2 \tilde{K}_h \Omega\tau - Q_1Q_o \tilde{K}_\alpha \mu r_\alpha^2\tau + \frac{2}{3}\mu Q_4^2 Q_1 \tilde{K}_h\Omega\tau \\
&- \frac{5}{27} Q_1Q_4^2 \frac{\tilde{K}_h\Omega}{M^2} + \frac{5}{9}\frac{\mu x_\alpha}{M}\tilde{K}_h\Omega Q_2Q_3Q_4 + \frac{1}{5}Q_2Q_3Q_4 \tilde{K}_h \Omega\tau^2 \\
&+ Q_1^2 Q_4 \mu r_\alpha^2 \tilde{K}_\alpha\tau + Q_1Q_4^2 \mu r_\alpha^2 \tilde{K}_h \Omega\tau - \frac{5}{9}Q_1^2 Q_4 r_\alpha^2 \frac{\tilde{K}_\alpha}{M^2} \\
&+ \frac{5}{3} Q_1Q_2Q_3\mu\frac{r_\alpha^2}{M} x_\alpha \tilde{K}_\alpha + \frac{3}{5}Q_1Q_2Q_3 r_\alpha^2 \tilde{K}_\alpha \tau^2 \\
&+ \frac{6}{5} Q_2Q_3Q_4 \mu x_\alpha \tilde{K}_h\Omega M\tau^2\Big] \\
&+ \Big[Q_1Q_4Q_o \mu r_\alpha^2 \tau - Q_2Q_3Q_o \mu x_\alpha^2 \tau - \frac{1}{3}Q_1^2 Q_4^2 \mu\tau \\
&+ \frac{5}{27}\frac{Q_1^2 Q_4^2}{M^2} - \frac{5}{9}Q_1Q_2Q_3Q_4 \frac{\mu x_\alpha}{M} - \frac{1}{5}Q_1Q_2Q_3Q_4 \tau^2 \\
&- Q_1^2 Q_4^2\mu r_\alpha^2\tau + \frac{5}{9}Q_1^2 Q_4^2 \frac{r_\alpha^2}{M^2} - \frac{5}{3}Q_1Q_2Q_3Q_4\frac{\mu r_\alpha^2 x_\alpha}{M}
\end{aligned} \tag{3.57}$$

$$- \frac{3}{5} Q_1 Q_2 Q_3 Q_4 \, r_\alpha^2 \, \tau^2 - \frac{6}{5} Q_1 Q_2 Q_3 Q_4 \, \mu x_\alpha \, M \, \tau^2$$

$$+ \frac{2}{3} Q_1 Q_2 Q_3 Q_4 \, \frac{x_\alpha \, \tau}{M} - 2 \, Q_2^2 \, Q_3^2 \, \mu x_\alpha^2 \, \tau$$

$$- \frac{18}{25} \, Q_2^2 \, Q_3^2 \, x_\alpha \, M \, \tau^3] = 0$$

The solution procedure is: Choose an initial guess M and solve Eq. (3.57) for $\lambda_1 \; \lambda_2$, the two roots. Only one of these will be of interest, the one corresponding to lowest M. Substitute λ_1 and λ_2 into Eq. (3.55) and compute k_1 and k_2. Then from Eq. (3.6)

$$M_1 = \frac{(C/2) \; \omega_1}{a_\infty \; k_1}$$

$$M_2 = \frac{(C/2)}{a_\infty} \; \frac{\omega_2}{k_2}$$

Take the lowest value (M_1 or M_2) and compare it with the initial guess value of M. If they agree, the problem is solved. If not use the new value of $M = M_1$ say ($M_1 < M_2$) and repeat the procedure. The criterion for convergence will be that the correct values of M, k, and λ satisfy equations (3.53) and (3.54) to within some prescribed small amount. This will guarantee that det $(Q^T A \, Q)$ is actually close to zero for what shall be the flutter condition.

Chapter IV

CRITERION FUNCTION

4.1 Energy

The basic criterion function in this study is to be the energy required for a sequence of flight conditions, the sum of which represent a mission. Therefore the quantity to be optimized may be written as

$$\Phi = \int_0^{t_M} D(t)\ U(t)\ dt \tag{4.1}$$

where D is the total drag, the sum of the pressure drag D_p and the friction drag D_f, U is the velocity ($U = Ma_\infty$), and t_M is the total time required for the mission.

A mission is, however, to be represented by a set of discrete flight conditions. This allows equation (4.1) to be recast as

$$\Phi = \sum_{i=1}^{S} D_i\ U_i\ t_i \tag{4.2}$$

where now

S = number of flight conditions

D_i = total drag in i^{th} flight condition

U_i = velocity in i^{th} flight condition

t_i = time in i^{th} flight condition and D_i, U_i, and t_i are assumed constant in any one flight condition.

4.2 Pressure Drag

The pressure drag of an airfoil may be found by integrating the horizontal component of the aerodynamic pressure differential over the surface of the airfoil. Then, for a strip of length $\frac{\ell}{N}$ with a total angle of attack $\bar{\alpha}_i = \alpha_i + \alpha_o$, the pressure drag may be written as

$$D_{p_i} = 2\gamma \; p_\infty \; M \left[\alpha_i + \alpha_o + \left(\frac{T}{C}\right)^2\right] C \frac{\ell}{N} \tag{4.3}$$

4.3 Friction Drag

The friction drag is defined[10] as

$$D_f = \iint \tau_f(x) \cos(\vec{t}, \vec{U}) \, dA \tag{4.4}$$

and

$\tau_f(x)$ = friction force per unit surface area of airfoil

$\vec{t}$ is a unit tangent vector to the surface

$\vec{U}$ is the free stream velocity vector.

Taking $\cos(\vec{t}, \vec{U}) \equiv \cos\varepsilon$, and considering the symmetric double wedge, ε has values in the first, second, third, and fourth quadrants, respectively, of

$$\begin{aligned}
\varepsilon_1 &= \theta + (\alpha_o + \alpha_i) \\
\varepsilon_2 &= \theta - (\alpha_o + \alpha_i) \\
\varepsilon_3 &= \theta + (\alpha_o + \alpha_i) \\
\varepsilon_4 &= \theta - (\alpha_o + \alpha_i)
\end{aligned} \tag{4.5}$$

where

$$\theta = \tan^{-1} \left(\frac{T}{C}\right) \tag{4.6}$$

Considering the integral over the surface in (4.4) to be independent of y over increments of length $\frac{\ell}{N}$,

$$D_{f_i} = \frac{\ell}{N} \int_s \tau_f(x) \cos [\vec{t}(x), \vec{U}] \; ds \tag{4.7}$$

and

$$ds = dx \, [1 + (T/C)^2]^{1/2} \tag{4.8}$$

Introducing (4.5) and (4.8) into (4.7) yields

$$D_{f_i} = \frac{2\,\ell}{N} \int_{-\frac{C}{2}}^{+\frac{C}{2}} \tau_f(x) \cos (\alpha_o + \alpha_i) \; dx \tag{4.9}$$

It is apparent from (4.9) that friction drag is not explicitly dependent on the thickness of the airfoil.

According to Nielsen[10], the frictional stress is given by

$$\tau_f(x) = \frac{C_F \rho_\infty^* U^2}{2} \tag{4.10}$$

$$C_F = \frac{0.370}{(\log_{10} Re^*)^{2.584}} \tag{4.11}$$

The super stars indicate that the quantities distinguished by them are to be evaluated at the so-called "reference temperature" T^*.

The Reynolds Number is given by

$$Re^* = \frac{U\,(x + \frac{C}{2})\,\rho_\infty^*}{\mu_a^*} \tag{4.12}$$

and also

ρ_∞^* = mass density of air at T^*, slugs/ft^3

μ_a^* = absolute viscosity at T^*, slugs/ft-sec.

In terms of (4.11) and (4.12), (4.10) may be expressed as

$$\tau_f(x) = \frac{0.370}{2}\,\rho_\infty^*\;U^2\;\left(\log_{10}\frac{U(x + \frac{C}{2})\,\rho_\infty^*}{\mu_a^*}\right)^{-2.584} \tag{4.13}$$

It is advisable to put (4.13) in a form dependent on fewer parameters. With this in mind, T^* will first be found in terms of T_∞, the free stream temperature, which will be known when the altitude is known. To begin, define

$$T_s = T_\infty \left(1 + \frac{\gamma + 1}{2} M^2\right) \qquad (4.14)$$

where T_s is the stagnation temperature. In addition introduce a quantity, T_w, which is the plate temperature at the prescribed Mach number. Then

$$T^* = T_\infty + \frac{1}{2}(T_w - T_\infty) + 0.22\ r\ (T_s - T_\infty) \qquad (4.15)$$

The quantity r is the "recovery factor" and is given by

$$r = \left(\frac{g\ \mu_a^*\ C_p^*}{k^*}\right)^{\frac{1}{3}} = \frac{T_R - T_\infty}{T_s - T_\infty} \qquad (4.16)$$

where

T_R = recovery temperature (wing equilibrium temperature)

g = acceleration due to gravity

C_p^* = specific heat of air at T^*

k^* = thermal conductivity of air at T^*

The recovery factor "r" is a measure of how close T_R approaches T_s, the free stream stagnation temperature.

For turbulent flow, r has been found to be approximately constant at

$$r = 0.90 \qquad (4.17)$$

and the ratio of specific heats, " γ ", is also approximately constant at 1.4.

One further assumption is that there is no heat loss from the plate due to reradiation. This means that

$$T_R = T_W \tag{4.18}$$

Again, the object of all this is to get equation (4.13) into a form which may be easily evaluated given the flight conditions imposed on the structure, i.e. M and T_∞ , because T_∞ is prescribed when the flight altitude is given. Now

$$Ma_\infty = U = (\gamma \; R \, T_\infty)^{1/2} \, M \tag{4.19}$$

$$R = 1718 \text{ ft}^2/\text{sec}^2{}^\circ\text{R} \tag{4.20}$$

and

$$\rho_\infty^* = \frac{p_\infty}{RT^*} \tag{4.21}$$

Ref. 13, by a process of curve fitting, has obtained an expression for μ_a^* as

$$\mu_a^* = \frac{0.2770 \times 10^{-7} \, T^{*\,3/2}}{(T^* + 198.7)} \tag{4.22}$$

Substitution of (4.20), (4.21), and (4.22) into (4.13) yields

$$\tau_f(x) = 0.259 \, p_\infty M^2 \frac{T_\infty}{T^*} \left[\log_{10} \frac{1.22\text{x}10^6 p_\infty T_\infty^{1/2} (T^*+198.7) M(x+ \frac{C}{2})}{T^{*\,5/2}}\right]^{-2.584} \tag{4.23}$$

To eliminate T^* from (4.23), use equations (4.15) and (4.16) with assumptions given in equations (4.17) and (4.18) to get, finally

$$\tau_f(x) = \frac{0.259 M^2 p_\infty}{(1+0.13M^2)} \left[\log_{10} \frac{1.22\times10^6 M p_\infty [T_\infty (1+0.13M^2)+198.7]}{T_\infty^{\;2} \, [1 + 0.13\, M^2\,]^{5/2}} (x+ \frac{C}{2})\right]^{-2.584} \tag{4.24}$$

Substitution of (4.24) into (4.9), and using a numerical integration technique, the friction drag per wing segment may be determined.

Chapter V

SYNTHESIS

5.1 Normalized Composite Behavior Function and Synthesis Method

The basic analysis for the system herein considered is contained in Chapters I through IV. Considering the behavior functions, root angle of attack (α_O) is given by equation (2.34), leading edge tip deflection (w_T) is given by equation (2.36), stress at the root (σ) is given by equation (2.50) and flutter Mach number is obtained by use of equations (3.6), (3.53), (3.54) (3.55), and (3.57). Also the principal criterion function (Φ), the total energy, is given by equation (4.2) and (4.3) and (4.9) and (4.24).

In a given design situation, each of the behavior functions, except flutter Mach number, will have a prescribed value which may not be exceeded. The flutter Mach number, on the other hand, must be greater than the actual flight Mach number. These prescribed values of the behavior functions will in general be different for each flight condition in the mission.

Consider that the number of flight conditions composing a mission is S. Then for each of these S flight conditions, it is advantageous to have the ability to prescribe different values in different flight conditions for each of the behavior functions. For instance, the root angle of attack will increase as the altitude increases and hence a larger acceptable value may be desirable at higher altitudes. Likewise, because of material fatigue, the allowable stress should be less in a flight condition of frequent occurence than in a flight condition which seldom occurs.

Each flight condition will then have prescribed values for each behavior function. The q^{th} flight condition would then be constrained by

1) maximum root angle of attack AMAX q
2) maximum leading edge tip deflection DMAX q
3) maximum root stress SMAX q
4) flutter Mach number MF q

In the case of the trade-off study, the maximum allowable wing weight, WMAX, will be included in the above list.

A simple way to handle these four behavior functions is to normalize them and to then construct a composite behavior function. Consider that an analysis has been completed and values of α_{oq}, w_{Tq}, σ_q and MF_q for $q = 1,2, \ldots, S$ have been obtained. Then using the prescribed quantities introduced above, define for the q^{th} flight condition,

$$\bar{\phi}_q = \text{Max}\left[\frac{(\alpha_o)\ q}{\text{AMAX } q}, \frac{(w_T)\ q}{\text{DMAX } q}, \frac{(\sigma)\ q}{\text{SMAX } q}, \frac{(M)\ q}{\text{MF } q} \right] \quad (5.1)$$

where the quantity $\bar{\phi}_q$ assumes the value of that ratio on the right of (5.1) which has the maximum value. Further, consider a function ϕ such that

$$\phi = \text{Max}\ [\ \bar{\phi}_1,\ \bar{\phi}_2\ ,\ \ldots,\ \bar{\phi}_q,\ \ldots,\ \bar{\phi}_S\ ] \qquad (5.2)$$

where again ϕ assumes the value of the maximum $\bar{\phi}_q$ on the right of (5.2). The quantity ϕ will be called the composite behavior function. A design for which

$$\phi \leq 1.0 \qquad (5.3)$$

is an acceptable design while a design for which

$$\phi > 1.0 \qquad (5.4)$$

is one in which one or more of the behavior constraints are violated. Obviously a critical design is one for which ϕ is very close to unity. Figure 7 shows the curve $\phi = 1$ for the test case synthesis of the following chapter.

One further ingredient is required for synthesis, and it is a method of selecting a new design once the acceptability (ϕ) and merit (Φ) of an initial design have been determined. For this study, the gradient-steep descent, alternate step method was selected, principally because the design variable space[3] is two dimensional (i.e. T and C are the design variable space coordinates) and therefore, only two alternate step directions exist. A graphical description and basic flow chart of this method are contained in Figures 5 and 6.

Consider that the criterion function $\Phi(\vec{x}^i)$ is known for an acceptable design point $\vec{x}^i$, where

$$\vec{x}^{i} = \begin{Bmatrix} T \\ \\ C \end{Bmatrix}^{i} \tag{5.5}$$

Then, for some new design point near the original point

$$\Phi(\vec{x}^{i+1}) \cong \Phi(\vec{x}^{i}) + (\vec{x}^{i+1} - \vec{x}^{i}) \nabla \Phi \Big|_{i} \tag{5.6}$$

The new design point $\vec{x}^{i+1}$ is to be reached from the old point by moving in the gradient direction such that the value of the criterion function assumes a more optimum value. Then

$$\vec{x}^{i+1} = \vec{x}^{i} + h \nabla \Phi \Big|_{i} \tag{5.7}$$

where h is a parameter that determines the length of the move. Comparing (5.6) and (5.7) it is found that

$$h \nabla \Phi \Big|_{i} = \frac{\Phi(\vec{x}^{i+1}) - \Phi(\vec{x}^{i})}{\nabla \Phi \Big|_{i}} \tag{5.8}$$

Now, defining a quantity which denotes the percentage change in Φ from the initial to the new point as

$$\Delta = \frac{\Phi(\vec{x}^{i+1}) - \Phi(\vec{x}^{i})}{\Phi(\vec{x}^{i})} \tag{5.9}$$

and then comparing (5.8) with (5.9), it is found that

$$h = \frac{\Delta \Phi(\vec{x}^{i})}{[\nabla \Phi|_{i}] \cdot [\nabla \Phi|_{i}]} \tag{5.10}$$

$$\vec{x}^{i+1} = \vec{x}^{i} + \Delta \left\{ \frac{\Phi(\vec{x})^{i}}{[\nabla \Phi|_i]\cdot[\nabla \Phi|_i]} \right\} \nabla \Phi|_i \qquad (5.11)$$

The quantity Δ as defined by (5.9) is a negative number

$$\Delta < 0 \qquad (5.12)$$

for the case when the move is made from an acceptable initial point to a new point. When the move is made from an unacceptable initial point,

$$\Delta > 0 \qquad (5.13)$$

The fundamental criterion function in this study is not differentiable. A finite difference approximation to the true gradient was therefore resorted to.

In two dimensions, the gradient is given by

$$\nabla\Phi\ (T,C) = \lim_{\substack{\Delta T\to 0\\ \Delta C\to 0}} \frac{\Phi\ (T+\Delta T,C) - \Phi(T,C)}{\Delta T}\vec{i} + \frac{\Phi\ (T,C+\Delta C)-\Phi\ (T,C)}{\Delta C}\vec{j} \qquad (5.14)$$

and the partial derivatives can be approximated by computing, for the T component

$$\frac{\Phi\ (T + \Delta T,C) - \Phi\ (T,C)}{\Delta T} \qquad (5.15)$$

and for the C component

$$\frac{\Phi\ (T,\ C + \ \Delta C) - \Phi(T,C)}{\Delta C} \qquad (5.16)$$

for progressively smaller values of ΔT and ΔC until their change from the previous calculation is less than some desired amount.

This will yield a very good approximation to the gradient as long as the curves of $\Phi(T,C)$ = constant are smooth. In order not to introduce any bias into the gradient components, the actual computation used

$$\frac{\Phi\ (T+\frac{1}{2}\,\Delta T,C)\ -\ \phi\ (T-\frac{1}{2}\,\Delta T,C)}{\Delta T} \tag{5.17}$$

for the T component and

$$\frac{(T,C+\frac{1}{2}\,\Delta C)\ -\ \Phi\ (T,C-\frac{1}{2}\,\Delta C)}{\Delta C} \tag{5.18}$$

for the C component.

Redesign points are located using (5.11) subject to (5.12) or (5.13). When finally a point is found at which ϕ takes on the value unity, alternate steps are taken tangent (Fig.5) to the constant criterion function (merit) curves, to determine if a point can be found for which $\phi < 1.0$. If such a point is found, a new gradient is computed at this point and moves are again made using (5.11). If, however, no point can be found for which $\phi < 1.0$, the present point at which ϕ is unity is called the optimum design and the synthesis is complete.

The alternate tangent steps described above use the components of the gradient. Since the constant Φ curves are not straight lines the points obtained in these alternate steps will lie on constant Φ curves of higher magnitude than the original point. An iterative process is required to get back on the same Φ curve that the alternate step was taken from. After this is done the point is checked to determine if it is acceptable or unacceptable.

5.2 Synopsis

In this section of this chapter, a statement of the problem will be given in terms of equation numbers and based on the format given in the introduction.

I. Given:

(A) the fixed parameters of the system:

1. span, ℓ, ρ_s
2. material density; ρ
3. required payload; L_p

(B) the prescribed flight conditions

1. Number of flight conditions; S
2. q^{th} flight condition parameters
   - (a) Altitude; ALT_q $(p_{\infty_q}, a_{\infty_q}, \rho_{\infty_q}, T_{\infty_q})$
   - (b) Mach number; M_q
   - (c) time in flight condition; t_q

(C) the prescribed behavior constraints for the q^{th} flight condition

1. maximum root angle of attack; AMAX q
2. maximum leading edge tip deflection; DMAX q
3. maximum root stress; SMAX q
4. flight Mach number; M_q (to be less than or equal to the flutter Mach number)
5. (trade-off study only) maximum wing weight; WMAX

(D) Appropriate side constraints on the design parameters, T and C such that

$$LBX \leq T \leq UBX$$

and

$$LBY \leq C \leq UBY$$

where LBX and LBY are lower limits of T and C imposed by fabrication techniques and kindred things while UBX and UBY are corresponding upper limits on T and C.

II. Determine:

The design variables, T and C such that the side constraints are not violated and the composite behavior function ϕ as defined by (5.2) satisfies (5.3) and where the individual parts of (5.2) are given by equations (5.1), (2.34), (2.36), (2.50), (3.6), (3.53), (3.54), (3.55), and (3.57), and finally that the criterion function Φ as given by (4.2), (4.3), (4.9) and (4.24) assumes an optimum value. Redesigns are made using (5.11) subject to (5.12) or (5.13).

5.3 Trade-off Study

In performing the trade-off study, the system will be optimized first with no upper limit on weight (i.e. WMAX taken sufficiently large to exert no influence on the optimum). This is the problem as stated in section 5.2, and yields an optimum design based on energy. The corresponding total airfoil weight is given by

$$WGT = \frac{1}{2}\rho g \ \ell \ TC \rho_S \tag{5.19}$$

where g is the acceleration due to gravity.

Next a reduced maximum allowable weight is specified such that WMAX is less than the weight (WGT) obtained for the energy optimum, and again the system is optimized based on energy. The penalty for this reduced weight is readily seen as an increase in the optimum energy.

In this manner a curve of optimum energy vs. allowable weight will be plotted. Subsequent investigation of this curve may result in a design more realistic than the optimum energy design would be.

Chapter VI

EXAMPLE SYNTHESES AND RESULTS

Several example systems were optimized using the completed computer program.

1.) Test Case

This is a system which has a mission consisting of one flight condition. Complete data are contained in Table 1, and Figure 7 shows the design path from the initial design to the optimum energy design point. The value associated with each merit contour in Figure 7 gives the magnitude of the total drag on the airfoil for design points on that contour. This was done by arbitrarily taking $U = t = 1$ in (4-2).

The lift drag ratio of the optimum design obtained is 4.6. Assuming the skin friction coefficient to be 0.003[10], Hilton[14] gives the maximum lift-drag ratio for a 5.9% double wedge airfoil flying at $M = 3.0$ as 5.6.

The flutter constraint is shown in Figure 7 as a straight line. This is true only in the region plotted in Figure 7. Also, the root angle of attack is a straight line in this region. The slight slope of the α_o = constant lines may be explained by considering the following: If the chord C is fixed, and the depth T is increased, the wing is made more rigid, thus necessitating a greater root angle of attack to compensate for the smaller elastic twist angle at each spanwise station.

2.) Production Run Case M1

The mission for this case contains three flight conditions. Data for these are to be found in Table 2 . The synthesis path is shown in Figure 8.

The optimum design is a 6.8% double wedge, whereas for the preceding test case it was a 5.9% double wedge.

It should be noted that only behavior constraints which at some point compose part of the composite constraint curve are shown in Figure 8. Root stress is, therefore, not shown.

3.) Trade-Off Study on Case M1

A plot of allowable airfoil weight (lbs.) vs. optimum energy (ft.-lbs.) is shown in Figure 9. The data points used to construct this curve are tabulated in Table 3. Table 3 also contains the values of the design and behavior variables associated with each point.

Run number (1) in Table 3 represents the absolute minimum weight design. No acceptable design is possible for a weight less than 3,015 lbs. Run number (7) represents the optimum design based on energy. The associated weight is quite high at 5,470 lbs. In run numbers (2) through (6) the weights listed are prescribed maximum weights, and the associated energies are the minimum energies possible at the corresponding prescribed maximum weights. The curve in Figure 9 may thus be used to obtain the energy associated with a design which has a prescribed maximum weight less than the weight obtained by pure energy optimization and greater than the absolute minimum weight.

4.) Production Run Case M2

This is essentially the same as Case M1. The behavior constraint limit on root stress is changed to make this constraint active. Table 4 contains the complete details of this case. It is seen (Table 4) that root stress in the second flight condition and flutter in the third are active.

The contours of energy, root angle of attack, flutter Mach number, leading edge tip deflection, and root stress contained in Figures 7 and 8 were obtained only for the purpose of illustrating the synthesis path. These contours were obtained by extensive gridding of the design space after the synthesis was complete and the region containing the optimum was known.

Chapter VII

CONCLUSIONS AND RECOMMENDATIONS

This study has demonstrated the feasibility of applying the synthesis concept to a system which has an aeroelastic technology. As itstands, the problem solved is admittedly of a highly idealized nature. Future effort may be directed to making the problem more realistic by consideration of the following:

1.) Formulate the wing as a plate rather than as a beam-type structure. In view of the optimum design obtained based on energy, this is a necessary change, because the chord length (C) is not small compared to the span (ℓ).

2.) Include the effects of a control system at the trailing edge.

3.) Use the skin thickness (d) as a design variable.

4.) Consider the wing to be tapered and/or swept.

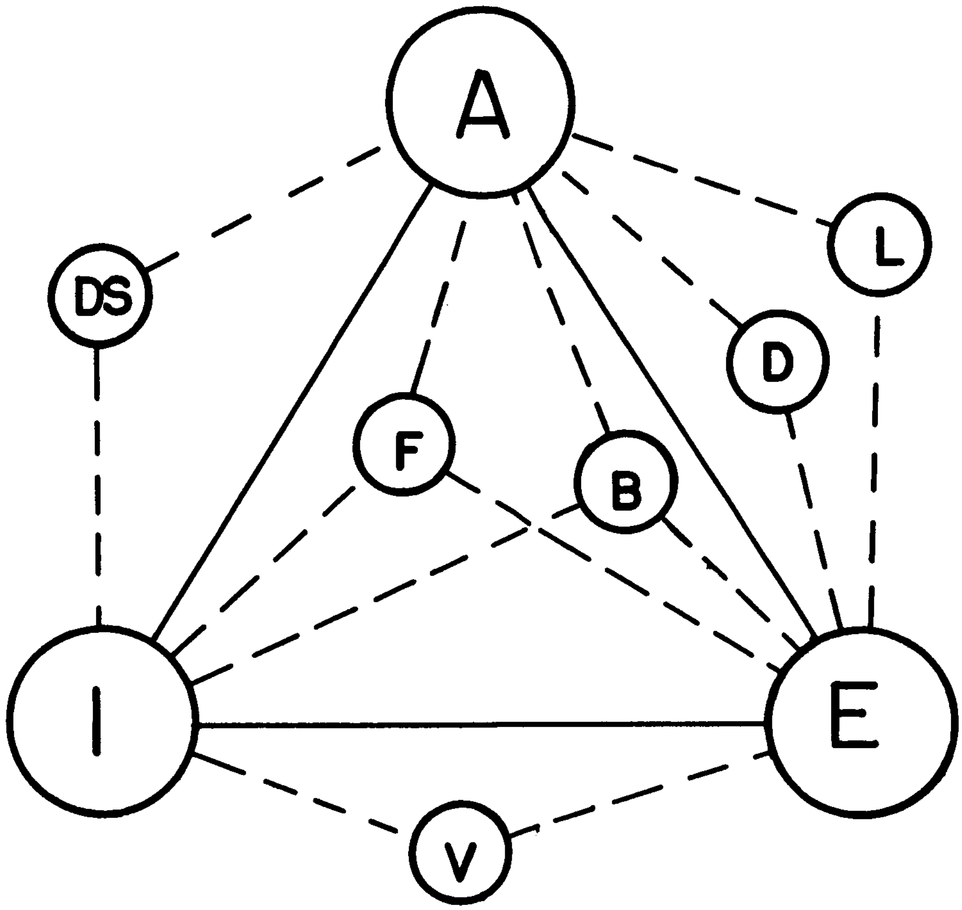

FIGURE 1

AERO ELASTIC TRIANGLE

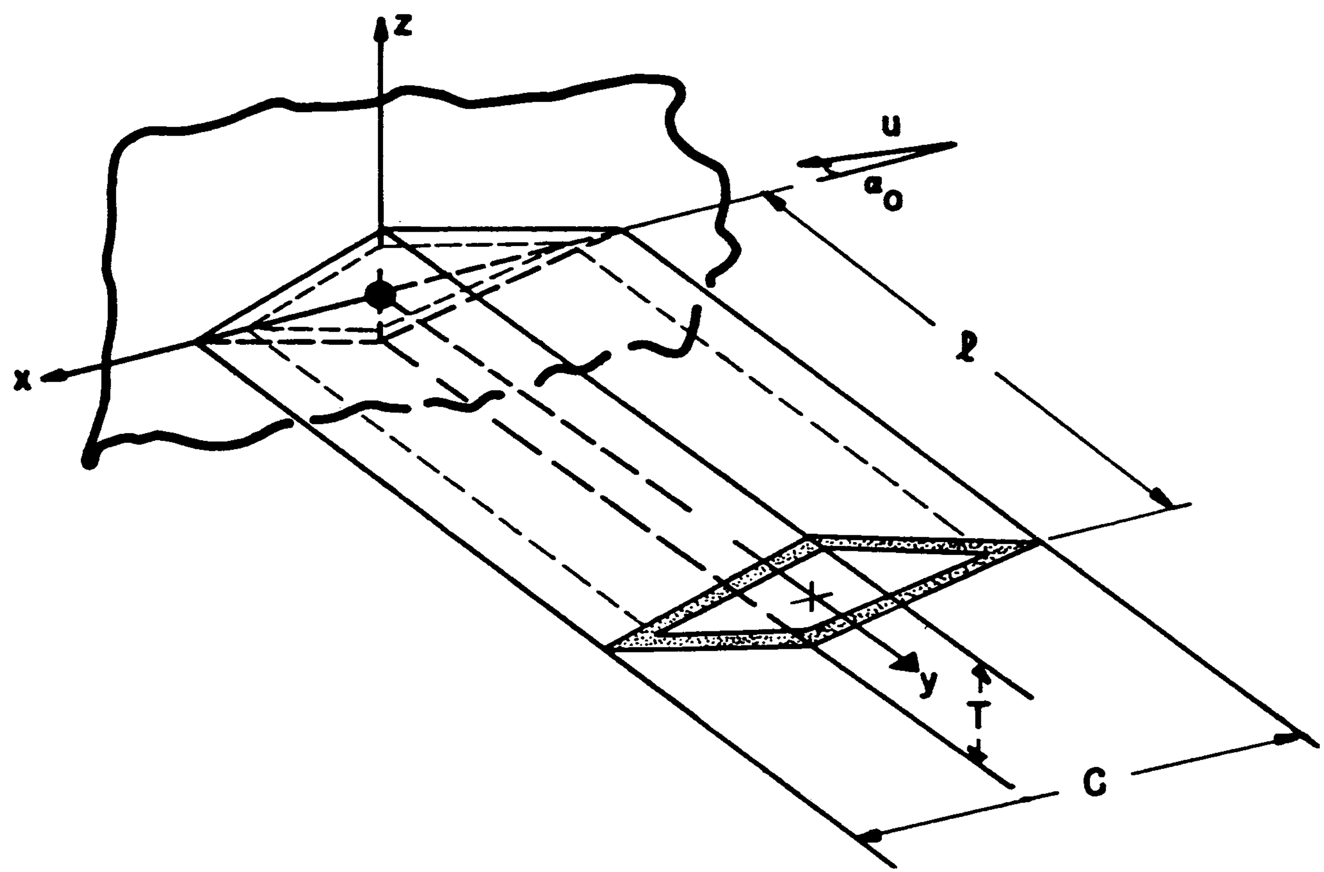

FIGURE 2
UNIFORM HOLLOW SYMMETRIC DOUBLE WEDGE AIRFOIL

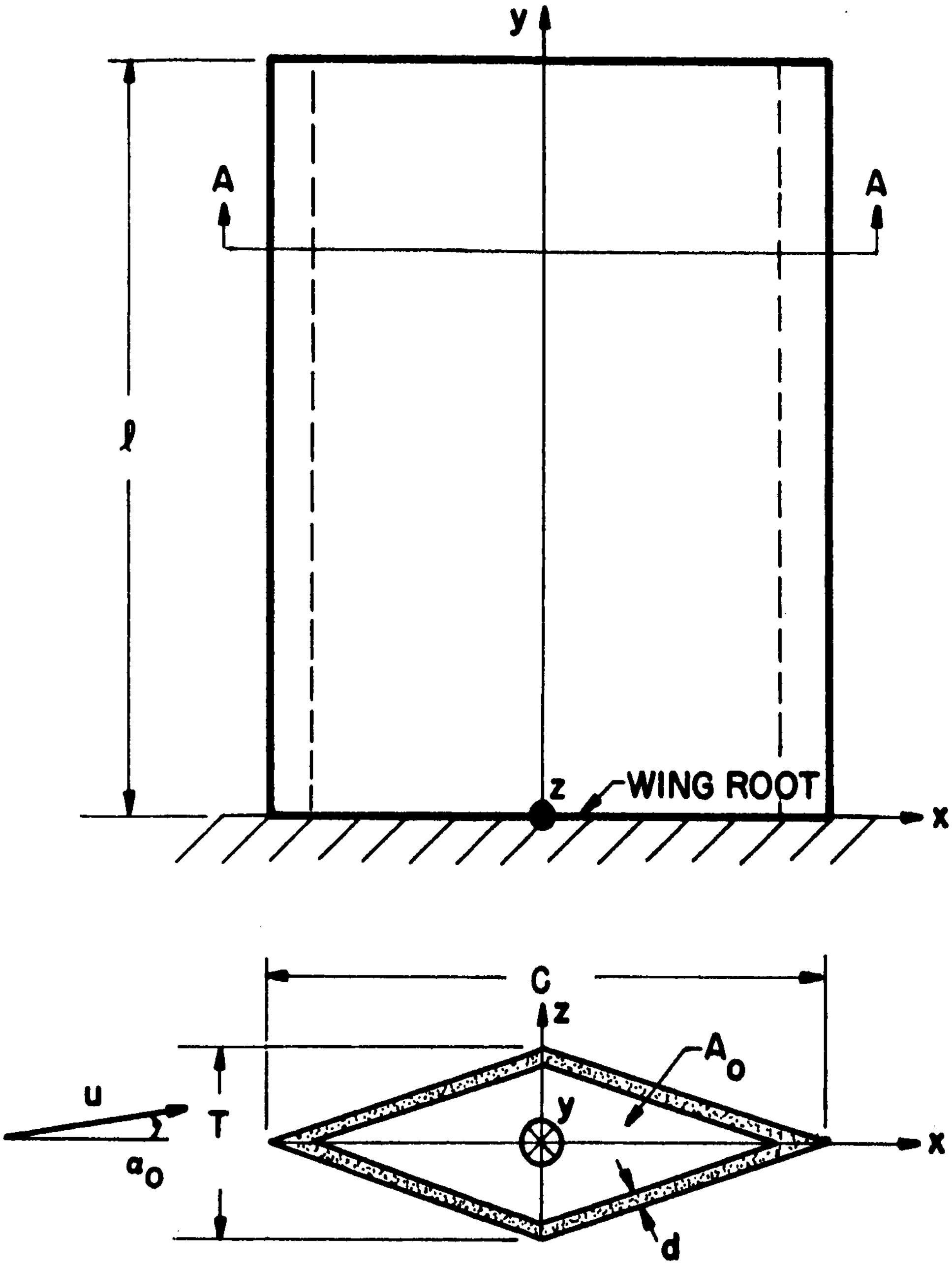

FIGURE 3A

WING PLANFORM AND CROSS SECTION AT ROOT

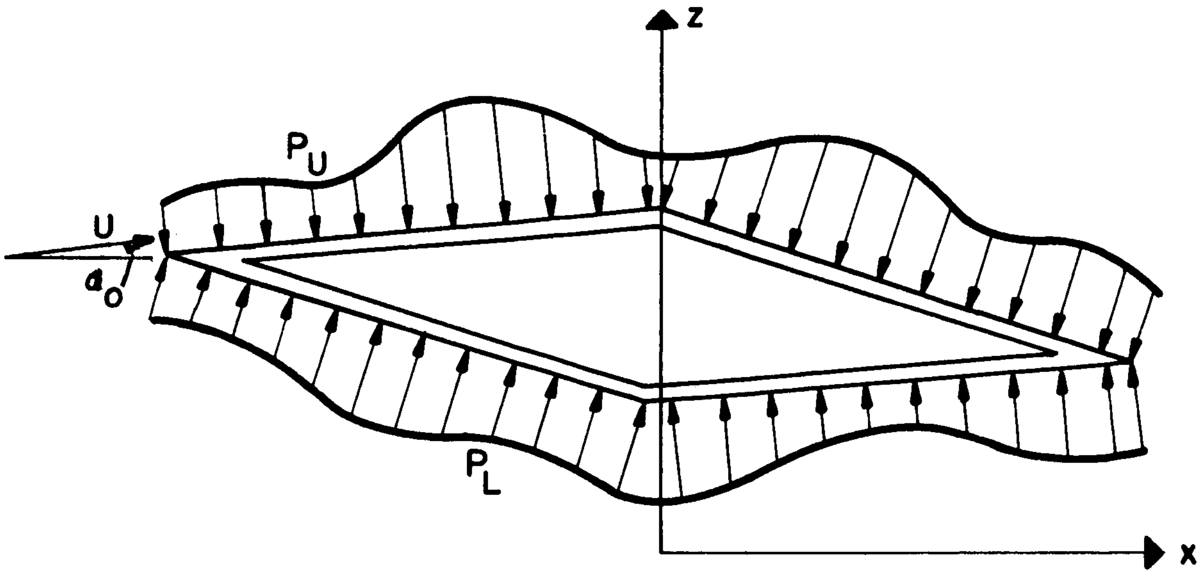

FIGURE 3B

PRESSURE DISTRIBUTION AT TYPICAL SECTION A-A

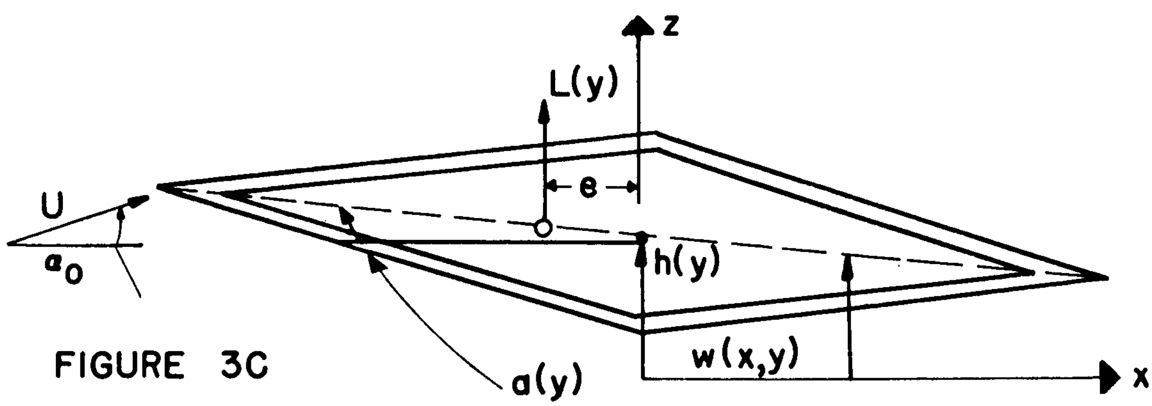

FIGURE 3C

DISPLACEMENT DISTRIBUTION AND RESULTANT PRESSURE (LIFT) PER UNIT SPAN AT TYPICAL SECTION A-A

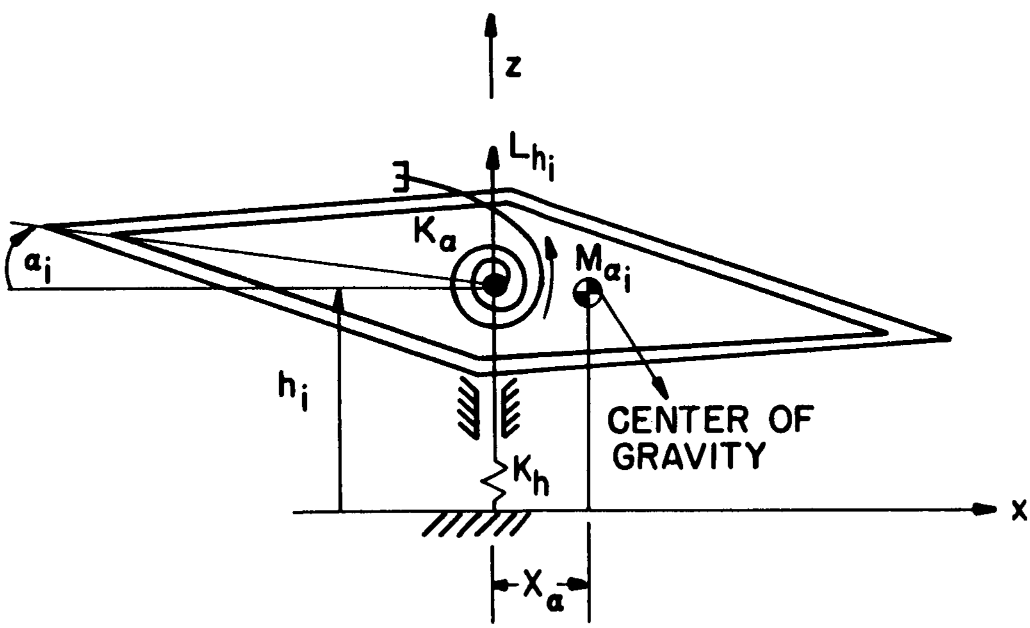

FIGURE 4

POSITIVE SENSE OF $\alpha_i, h_i, L_{h_i},$ & M_{α_i}

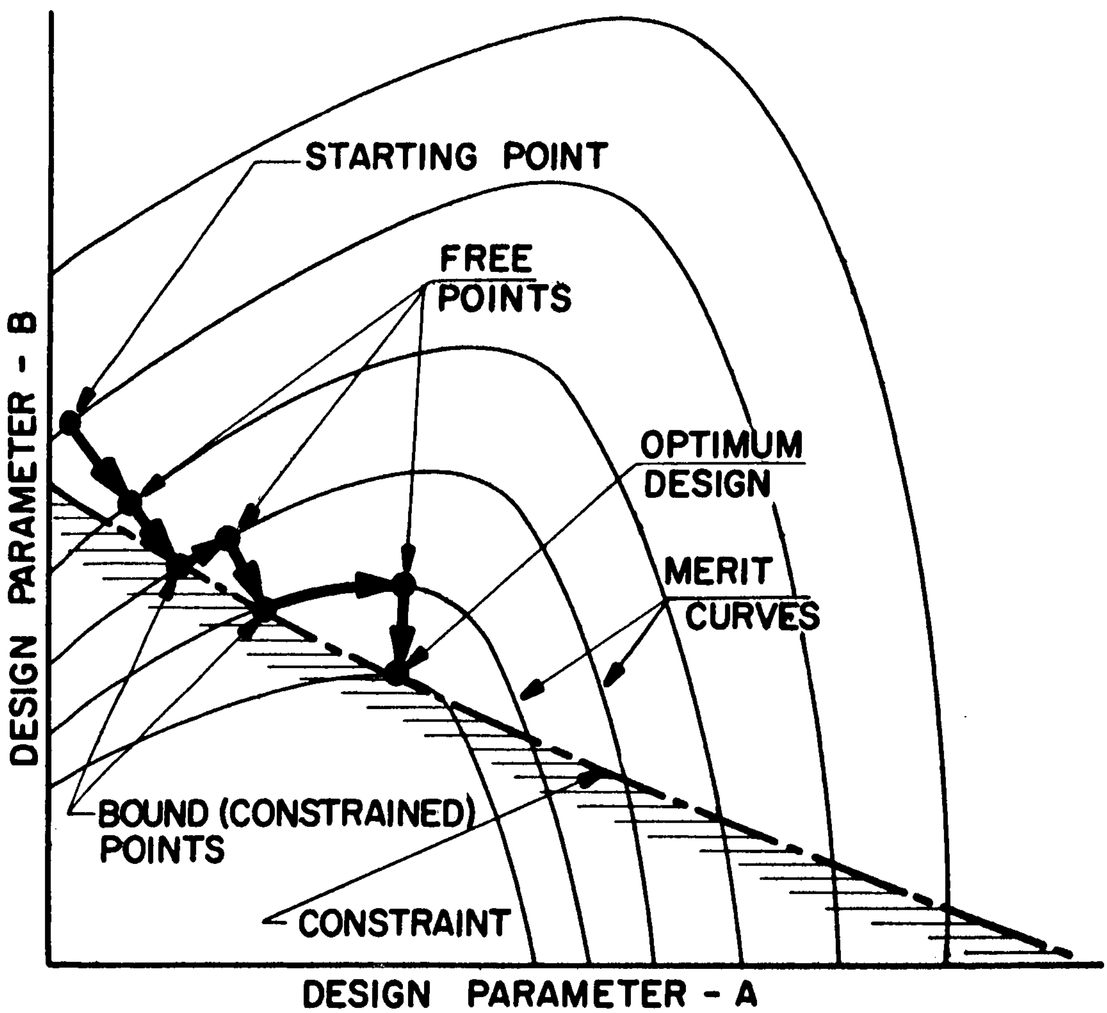

FIGURE 5 THE GRADIENT ALTERNATE STEP METHOD

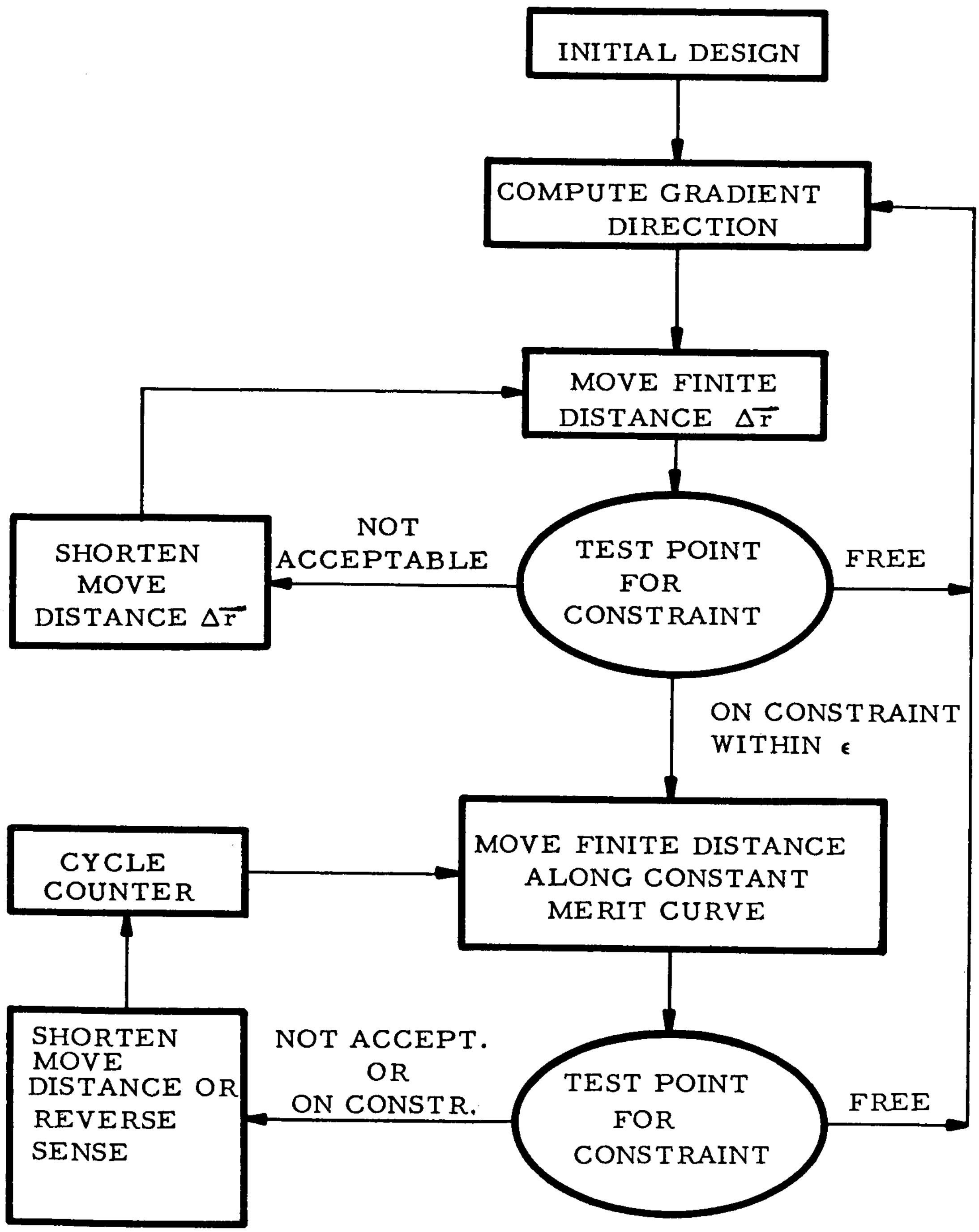

FIGURE 6 BASIC FLOW DIAGRAM FOR STEEP DESCENT-ALTERNATE STEP

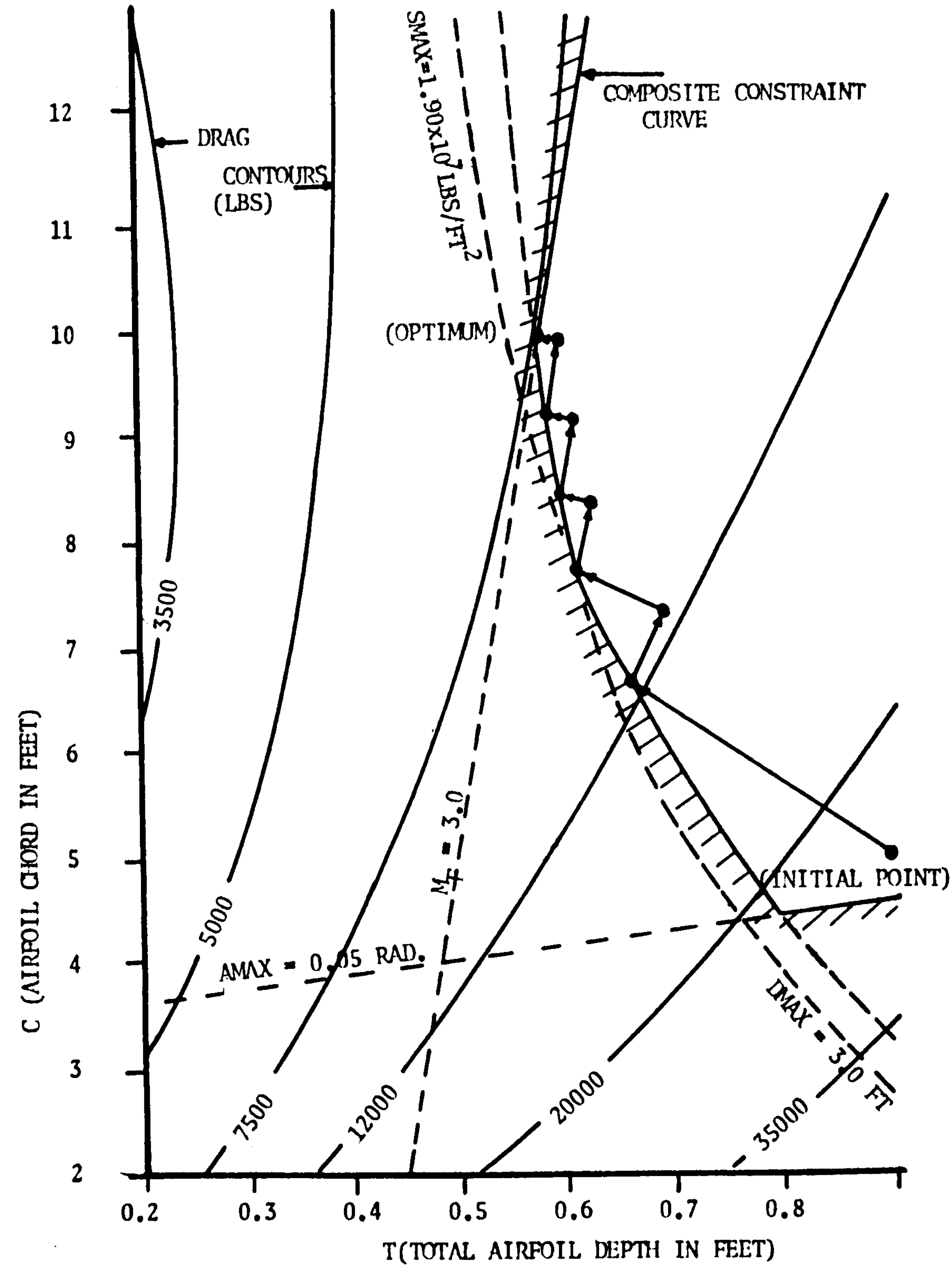

FIGURE 7 DESIGN PATH FOR TEST CASE

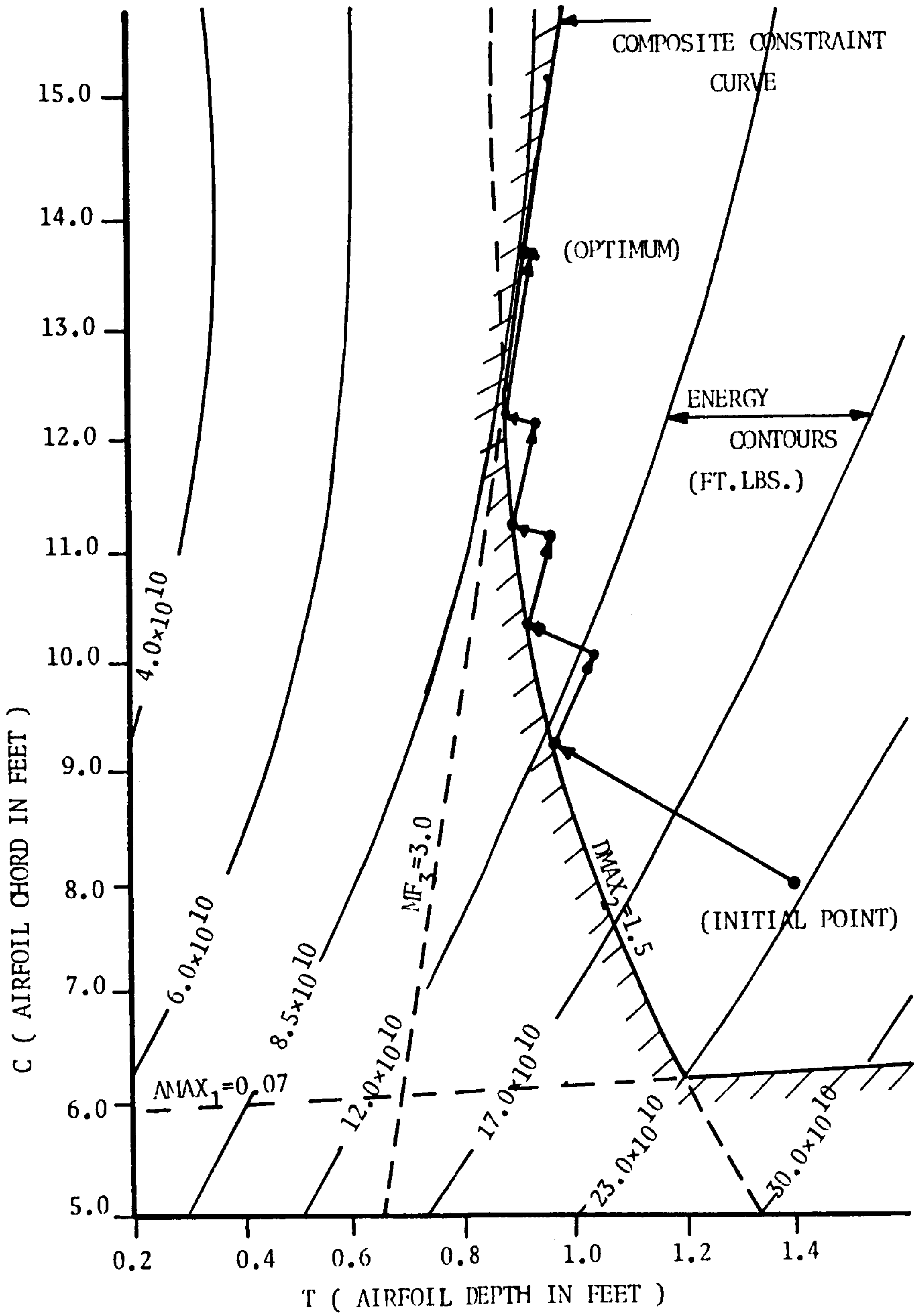

FIGURE 8 DESIGN PATH FOR CASE M1

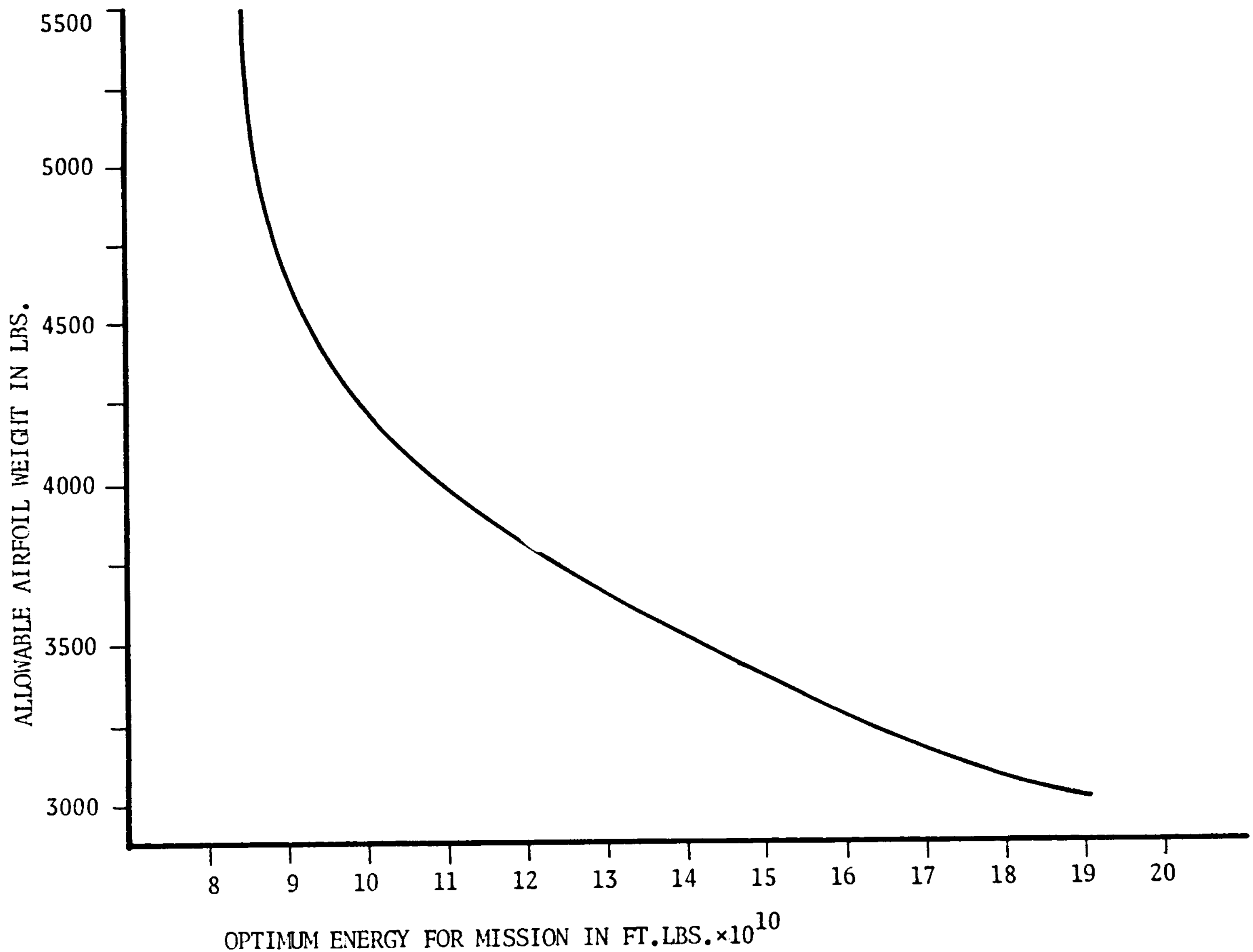

FIGURE 9 TRADE-OFF OF WEIGHT AND ENERGY ON CASE M1

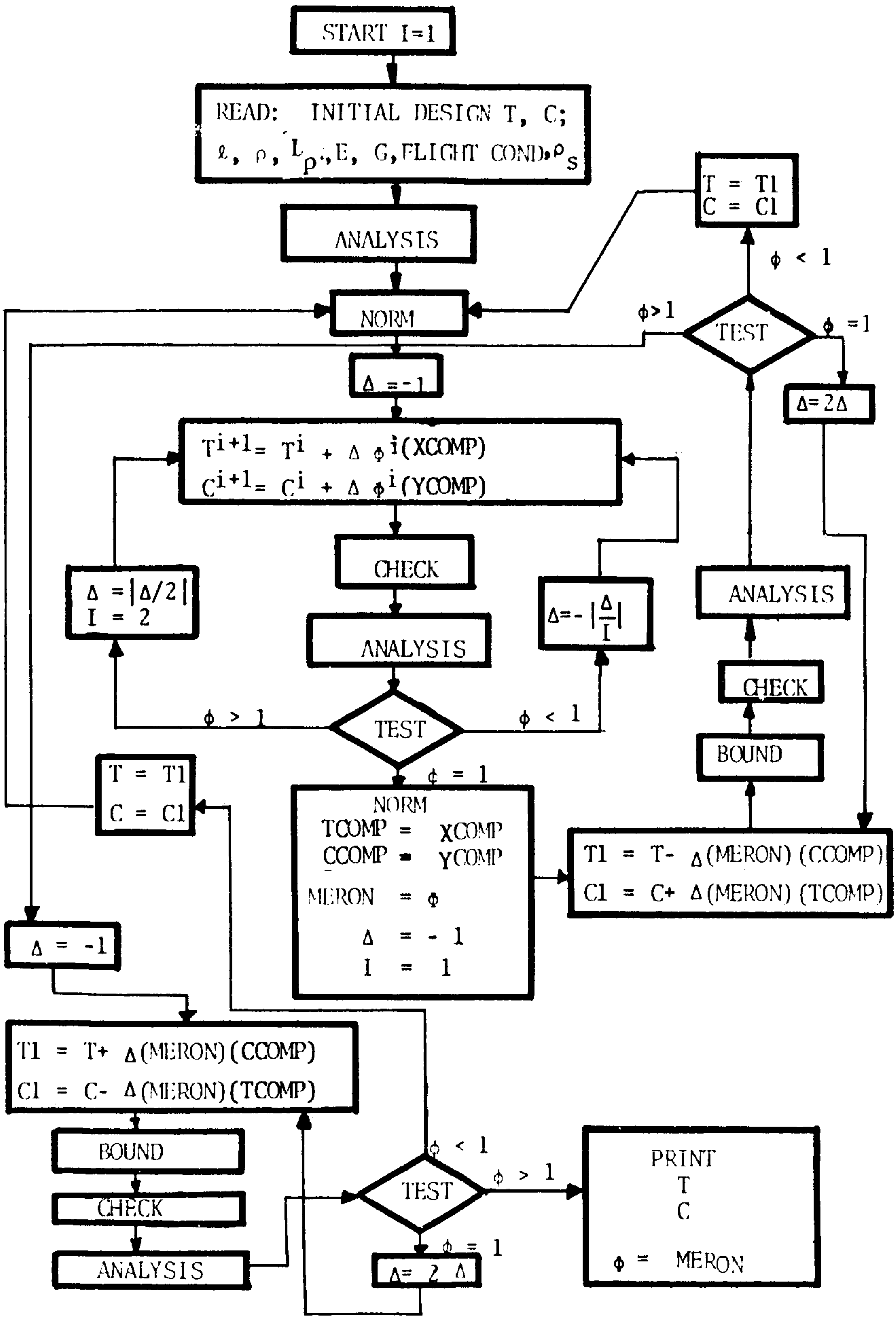

FIGURE 10 BLOCK DIAGRAM FOR COMPUTER PROGRAM

FIXED PARAMETERS	span	Lift(L_p)	Material	ρ	E	G	ρ_s
	30.0	30,000	Steel	15.4	43.2×10^8	17.3×10^8	0.1

SIDE CONSTRAINTS	LBX	LBY	UBX	UBY
	0.1	3.0	2.0	15.0

FLIGHT CONDITIONS								BEHAVIOR CONSTRAINT LIMITS			
FC#	Alt.	p_∞	a_∞	ρ_∞	T_∞	M	t	SMAX	DMAX	AMAX	M
1	30,000	500.0	971.1	8.9×10^{-4}	411.1	3.0	1.0	1.9×10^7	3.0	0.05	3.0

SYNTHESIS RESULTS									
INITIAL DESIGN					FINAL DESIGN				
T	C	d	ENERGY	WEIGHT	T	C	d	ENERGY	WEIGHT
0.9	5.0	0.023	23,235	3336	0.587	9.99	0.015	7544	4355
FC#	σ	w_T	α_o	M_F	FC#	σ	w_T	α_o	M_F
1	1.4×10^7	1.64	0.045	6.43	1	1.71×10^7	2.98*	0.021	3.04*

*active constraint

TABLE 1 TEST CASE

NOTE: Units for all quantities are as given in the list of Symbols.

FIXED PARAMETERS	Span	Lift(L_p)	Material	ρ	E	G	ρ_s
	30.0	30,000	Titanium	8.7	23.0×10^8	9.2×10^8	0.1

SIDE CONSTRAINTS	LBX	LBY	UBX	UBY
	0.1	3.0	2.0	15.0

FLIGHT CONDITIONS								BEHAVIOR CONSTRAINT LIMITS			
FC#	Alt.	p_∞	a_∞	ρ_∞	T_∞	M	t	SMAX	DMAX	AMAX	M
1	70,000	151.0	971.1	2.24×10^{-4}	392.4	5.0	600	2.75×10^7	2.0	0.07	5.0
2	50,000	243.0	971.1	2.24×10^{-4}	392.4	2.5	3600	1.60×10^7	1.5	0.09	2.5
3	30,000	628.0	994.4	8.89×10^{-4}	411.4	3.0	300	2.75×10^7	2.0	0.05	3.0

SYNTHESIS RESULTS

INITIAL DESIGN					FINAL DESIGN				
T	C	d	ENERGY	WEIGHT	T	C	d	ENERGY	WEIGHT
1.4	8.0	0.036	22.7×10^{10}	4761	0.937	13.73	0.024	8.50×10^{10}	5470
FC#	σ	w_T	α_o	M_F	FC#	σ	w_T	α_o	M_F
1	3.61×10^6	0.71	0.057	16.53	1	4.88×10^6	1.19	0.032	8.21
2	3.58×10^6	0.76	0.072	12.64	2	4.73×10^6	1.21	0.041	5.95
3	3.63×10^6	0.57	0.022	7.31	3	4.78×10^6	1.06	0.012	3.01*

*active constraint

TABLE 2 PRODUCTION RUN CASE M1

NOTE: Units for all quantities are as given in the list of symbols.

RUN NUMBER	WEIGHT (LBS)	ENERGY FT.LBS×10^{10}	DESIGN VARIABLES		F.C. NUMBER	BEHAVIOR VARIABLES			
			T(FT)	C(FT)		w_T(FT)	σ(LBS/FT^2)	α_o(RAD.)	M_F
1	3015	19.1	1.11	6.40	1	1.44	7.24×10^6	0.069*	12.83
					2	1.49*	7.15×10^6	0.089*	9.77
					3	1.30	7.29×10^6	0.027	5.60
2	3200	16.5	1.07	7.06	1	1.45	7.07×10^6	0.063	12.14
					2	1.49*	6.99×10^6	0.083	9.21
					3	1.32	7.13×10^6	0.024	5.22
3	3500	14.0	1.03	7.99	1	1.46	6.83×10^6	0.050	11.29
					2	1.49*	6.74×10^6	0.071	8.52
					3	1.34	6.93×10^6	0.021	4.76
4	3750	12.3	0.99	8.87	1	1.45	6.57×10^6	0.049	10.60
					2	1.49*	6.48×10^6	0.064	7.95
					3	1.33	6.67×10^6	0.019	4.37
5	4250	9.9	0.93	10.71	1	1.47	6.32×10^6	0.041	9.18
					2	1.49*	6.10×10^6	0.052	6.79
					3	1.34	6.15×10^6	0.015	3.60
6	5000	8.6	0.91	12.97	1	1.32	5.45×10^6	0.033	8.16
					2	1.35	5.25×10^6	0.043	5.93
					3	1.20	5.30×10^6	0.013	3.01*
7	5470	8.5	0.94	13.73	1	1.19	4.88×10^6	0.032	8.21
					2	1.21	4.73×10^6	0.041	5.95
					3	1.06	4.78×10^6	0.012	3.01*

*active constraint

TABLE 3 DATA FOR TRADE-OFF STUDY ON CASE M1

FIXED PARAMETERS	Span	Lift(L_p)	Material	ρ	E	G	ρ_s
	30.0	30,000	Titanium	8.7	23.0×10^{8}	9.2×10^{8}	0.1

SIDE CONSTRAINTS	LBX	LBY	UBX	UBY
	0.1	3.0	2.0	15.0

FLIGHT CONDITIONS								BEHAVIOR CONSTRAINT LIMITS			
FC#	Alt.	p_∞	a_∞	ρ_∞	T_∞	M	t	SMAX	DMAX	AMAX	M
1	70,000	151.0	971.1	2.24×10^{-4}	392.4	5.0	600	4.5×10^{6}	2.0	0.07	5.0
2	50,000	243.0	971.1	2.24×10^{-4}	392.4	2.5	3600	4.25×10^{6}	1.5	0.09	2.5
3	30,000	628.0	994.4	8.89×10^{-4}	411.4	3.0	300	4.5×10^{6}	2.0	0.05	3.0

SYNTHESIS RESULTS

INITIAL DESIGN					FINAL DESIGN				
T	C	d	ENERGY	WEIGHT	T	C	d	ENERGY	WEIGHT
1.4	8.0	0.036	22.7×10^{10}	4761	0.965	14.54	0.025	8.55×10^{10}	5966
FC#	σ	w_T	α_o	M_F	FC#	σ	w_T	α_o	M_F
1	3.61×10^{6}	0.71	0.057	16.53	1	4.29×10^{6}	1.06	0.030	8.31
2	3.58×10^{6}	0.76	0.072	12.64	2	4.21×10^{6}*	1.09	0.038	6.01
3	3.63×10^{6}	0.57	0.022	7.31	3	4.34×10^{6}	0.92	0.011	3.02*

TABLE 4: PRODUCTION RUN CASE M2

*active constraint

NOTE: Units for all quantities are as given in the list of symbols.

TABLE 5 Program Variable Names

AMAX(S)	upper limit on α_o in S^{th} flight condition
ANALI(X,Y)	procedure which determines M_F,
ATA(S)	root angle of attack
BOUND	prevents diverging alternate step
BSC(X,Y)	normalization procedure for $C_{h_{ij}}$
C, Y	airfoil chord
CCOMP	gradient component in C direction at a bound point (ϕ=1)
CHB(X,Y)	bending frequency normalization procedure
CHECK	tests side constraint violation
CHT()	torsional frequency normalization procedure
CONSTRAINT(X,Y)	composite constraint function (ϕ) procedure
DMAX(S)	upper limit on w_T
E	Young's Modulus
ENG(X,Y)	procedure for determining Φ , the total energy
EPS	error parameter for TEST( )
EPN	error parameter for ENG( )
FR	frequency ratio Ω
G	shear modulus
L, LO	number of flight conditions
LB	ω_h
LF	L_T total lift
LT	ω_α

MAR	energy per flight condition procedure
MAS(X,Y)	wing segment mass
MCH(S)	flutter Mach number in S^{th} flight condition
MOMNT	M_t
N, NO	number of wing segments
NORM(X,Y)	gradient routine
NRMBD	$\vec{e}_h$
NRMTR	$\vec{e}_\alpha$
PER	move size control parameter
PDRAG(X,Y,S)	procedure for w_T, α_o, $\vec{\alpha}$, σ, D_p
PRES(S)	pressure p_∞
RHO	airfoil material density (lbs/in^3)
RHOA(S)	air density ρ_∞
S	flight condition specifier
SKNFN(X,Y)	skin friction stress procedure τ_f
SMAX(S)	upper limit on σ
SPAN	ℓ
STR(S)	root stress σ
T1, C1	design variables after a tangent move
T, X	total airfoil depth
TCOMP	gradient component in T direction at a bound point
TEMP(S)	free stream temp. T_∞
TEST(X,Y)	tests ϕ and determines appropriate move
TIME(S)	flight time in S^{th} flight condition

TSC(X,Y)	normalization procedure for $C_{\alpha_{ij}}$
VAIR($\dot{S}$)	free stream velocity of sound a_∞
VEL($\dot{S}$)	Ma_∞
WGT(X,Y)	total wing weight
XCOMP	gradient component in T direction
YCOMP	gradient component in C direction

REFERENCES

1. Schmit, L. A., and Mallett, R.H., "Structural Synthesis and the Design Parameter Hierarchy," J. Structural Div., Proc. Am. Soc. Civil Engrs., 89, No. ST4, pp. 269-299, 1963.

2. Schmit, L. A., Kicher, T.P., and Morrow, W.M., "Structural Synthesis Capability for Integrally Stiffened Waffle Plates", AIAA Journal, Vol. 1, No. 12, pp. 2820-2836, 1964.

3. Schmit, L. A., and Fox, R. L., "Synthesis of a Simple Shock Isolator," NASA CR 55, 1964.

4. Collar, A. R., "The Expanding Domain of Aeroelasticity," Journal of the Royal Aeronautical Society, Vol. I, pp. 613-636, 1946.

5. Ashley, H., and Zartarian, G., "Piston Theory - A New Aerodynamic Tool for the Aeroelastician," Journal of the Aeronautical Sciences, Vol. 23, pp. 1108-1118, 1956.

6. Mills, W. R., "Procedures for Including Temperature Effects in Structural Analyses of Elastic Wings - Part I" WADC Tech. Rept. 57-754, 1958.

7. Roark, R. J., "Formulas for Stress and Strain," McGraw-Hill Book Co. p. 177, 1954.

8. Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., "Aeroelasticity," Addison Wesley, pp. 38-47, 1955.

9. Crandal, S.H., and Dahl, N.C., "An Introduction to the Mechanics of Solids," McGraw-Hill, pp. 266-269, 1959.

10. Nielsen, J. N., "Missile Aerodynamics", McGraw-Hill, pp. 261-348, 1960.

11. Scanlan, R.H. and Rosenbaum, R., "Introduction to the Study of Aircraft Vibration and Flutter," Macmillan, pp. 192-204, 1951.

12. Bisplinghoff, R.L., Op.cit., pp. 527-631.

13. Bloom, M., "Thermal Conditions associated with Aircraft in Flight," WADC Tech. Rept. 55-169, 1956.

14. Hilton, W. F., "High Speed Aerodynamics," Longman's-Green, pp. 192-205, 1951.

15. Bottenbruch, H., "Structure and Use of Algol 60," Journal of the Association for Computing Machinery, Vol. 9, No. 2, 1962.

APPENDIX

The computer program was written in the Algol 60 Compiler[15] for the Univac 1107 Electronic Computer. This appendix contains an explanation of the program, a table of program variable names, and a sample program listing.

Extensive use is made of the procedure routines of this compiler. A procedure is an independent subprogram with specified formal parameters. Procedure names may be assigned numerical values by designating the procedure as a "real" procedure.

Eight basic procedures are used in the completed program. They are ANAL1, PDRAG, ENERGY, NORM, CONSTRAINT, TEST, BOUND, and CHECK. These and others are listed in Table 5. The basic procedures are discussed below:

1) PROCEDURE ANAL1(X,Y)

This procedure computes the flutter Mach number. The formal parameters shown in the procedure heading above correspond to the actual design variables T and C respectively. When the procedure is called, X is replaced by the current value of T, and Y is replaced by the current value of C.

In order to compute the flutter Mach number, certain preliminary calculations must be made. First, the flexibility influence coefficients, $C_{\alpha_{ij}}$ and $C_{h_{ij}}$ are computed using (2.17) and (2.21). Then the fundamental uncoupled bending and torsion mode shapes ($\vec{e}_h$, $\vec{e}_\alpha$) and frequencies (ω_h, ω_α) are computed using a matrix iteration technique on (3.40) and (3.41). Next, the matrices $C_{h_{ij}}$ and $C_{\alpha_{ij}}$ are inverted to obtain the stiffness matrices $K_{h_{ij}}$ and $K_{\alpha_{ij}}$. These are then normalized as indicated in (3.30) and (3.31) to give $\tilde{K}_{h_{ij}}$ and $\tilde{K}_{\alpha_{ij}}$. Finally the matrix multiplications indicated in (3.49) are performed to give $\tilde{K}_h$, $\tilde{K}_\alpha$, Q_1, Q_2, Q_3, and Q_4. These preliminary results are then used along with equations (3.6), (3.53), (3.54), and (3.57) to evaluate the flutter Mach number for each flight condition.

2) REAL PROCEDURE PDRAG (X,Y,S)

This procedure computes α_o, $\vec{\alpha}$, $\vec{w}$, $\vec{M}_t$, σ, and D_p. The pertinent equations are (2.32), (2.27), (2.34), (2.15), (2.50) and (4.3). The formal parameter "S" in the heading specifies the flight condition to which the above results pertain.

The value D_p is assigned to the name of the procedure as

$$\text{PDRAG} = D_p \tag{A.1}$$

for each succeeding flight condition.

3) REAL PROCEDURE ENG(X,Y)

This procedure is essentially a Simpson's Rule integration routine which is necessary to evaluate the integral in (4.9) and thus to determine the friction drag. Let the evaluated integral for the s^{th} flight condition be denoted by ψ_s. Then introduce a real procedure MAR such that the argument of MAR is

$$\text{MAR}(T,C,\psi_s,s) = [\text{PDRAG}(T,C,s) + \frac{2\ell}{N}\psi_s]\,(U_s)(t_s) \tag{A.2}$$

so that given T, C, and ψ_s, MAR yields the total energy to complete the s^{th} flight condition. More than one flight condition necessitates repeating (A.2) for each one.

In the general case when (A.2) must be repeated for S flight conditions, the total energy required to accomplish these S flight conditions (Φ) is assigned to the procedure name ENG as

$$\Phi = \mathrm{ENG}(T,C) = \sum_{s=1}^{S} \mathrm{MAR}\ (T,\ C,\ \psi_s,\ s) \qquad \text{(A.3)}$$

4) PROCEDURE NORM(X,Y)

This is a routine for computing the components of the gradient to the constant merit curves. These gradient components are then normalized to give move direction components as

$$\mathrm{XCOMP} = \frac{\Phi,_T}{[\nabla\ \Phi].[\nabla\ \Phi]} \qquad \text{(A.5)}$$

$$\mathrm{YCOMP} = \frac{\Phi,_C}{[\nabla\ \Phi].[\nabla\ \Phi]} \qquad \text{(A.6)}$$

where the comma indicates differentiation.

5) REAL PROCEDURE CONSTRAINT(X,Y)

This procedure computes the value of the composite constraint function as given by (5.2) and assigns this value to the procedure name CONSTRAINT. It is used in conjunction with procedure TEST to determine the appropriate move direction.

6) PROCEDURE TEST (X,Y,ON,FREE,NOT)

This routine determines if a design is acceptable by computing the composite constraint function ϕ . The code name for ϕ is CONSTRAINT(X,Y).

ON, FREE, and NOT are the program move locations of the three possible situations, as

$$\begin{aligned} &\text{ON} \;\;\;\; ; \; \phi = 1 \\ &\text{FREE} ; \; \phi < 1 \\ &\text{NOT} \;\; ; \; \phi > 1 \end{aligned} \tag{A.7}$$

7) PROCEDURE BOUND (X,Y, XCOMP, YCOMP)

This procedure is used after an alternate step to get back on a merit contour of the same value as that from which the move was taken.

The values MERON, TCOMP, and CCOMP are the stored values of ϕ, XCOMP, and YCOMP corresponding to the bound point (ϕ=1) from which the alternate step move was taken.

8) PROCEDURE CHECK (X,Y, XCOMP, YCOMP, XS, YS)

This checks the values of the design variables against those of the side constraints after a move is taken but before any part of the analysis is undertaken. If a side constraint is violated, the move distance is reduced until it is not. XS and YS are move direction control integers which take on values of +1 or -1 depending on where the move was taken from and the type of move made, i.e., gradient direction or tangent direction.

A block diagram using the procedures and symbols defined above is given in Fig. 10. In Fig. 10, the procedures ANAL1, PDRAG, and ENG will be grouped together under the name ANALYSIS as

$$\text{ANALYSIS} \equiv \begin{Bmatrix} \text{ANAL1} \\ \text{PDRAG} \\ \text{ENG} \end{Bmatrix} \tag{A.8}$$

for ease of presentation.

The computer program symbols with explanations are listed in Table 5. The appendix is concluded with a complete sample program listing.

```
MONITOR SYSTEM -- 65KB3 08/08/64
    RUN     ENGINEERING C DIVISION

  ALC EROSYN
     ALGOL      JULY 11,1964          INTERFACE   JULY  6,  1964       PASS2   APRIL 19, 1964
000001      1      COMMENT AEROELASTIC OPTIMIZATION PROGRAM $                                      B0
000001      2     BEGIN
000001      3      INTEGER I,J,K,P,G,R,S,N0,V0,L0          $                                        B1
BLOCK 1     LEVEL 1
000005      4      REAL ARRAY NRMBD(1..5),NRMTR(1..5),DFL(1..5),
000024      5      STR(1..5),ATA(1..5),MCH(1..5),X0(1..5),LF(1..5),
000055      6      PRES(1..5),TEMP(1..5),VAIR(1..5),TIME(1..5),
000101      7      VEL(1..5),RHOA(1..5),MACH(1..5),DMAX(1..5),SMAX(1..5),AMAX(1..5)     $
000137      8      REAL T,C,SPAN,E,RHO,G,WMAX,LBX,LBY,UBX,UBY,EPS,EPN,EPSILON,
000137      9     MERIT,MERON,T1,C1,FER,XCOMP,YCOMP,TCOMP,CCOMP,TS,CS,
000137     10      EX,EY,ETA,SR,DT,TAU            $
000137     11     LOCAL LABEL ON,FREE,NO,N1,F1,NO1,F2,ON1,NOT1,ON2,NOT2,NOT3,DONE         $
000137     12        LIST RD(N0,V0,L0,FOR I=(1,1,V0)DO X0(I),
000141     13     FOR I=(1,1,L0)DO LF(I),
000141     14       FOR I=(1,1,L0)DO PRES(I),
000141     15     FOR I=(1,1,L0)DO MACH(I),
000141     16     FOR I=(1,1,L0)DO TEMP(I),
000141     17     FOR I=(1,1,L0)DO VAIR(I),
000141     18     FOR I=(1,1,L0)DO RHOA(I),
000141     19     FOR I=(1,1,L0)DO TIME(I),
000141     20     FOR I=(1,1,L0)DO VEL(I),
000141     21      FOR I=(1,1,L0)DO DMAX(I),
000141     22     FOR I=(1,1,L0)DO SMAX(I),
000141     23      FOR I=(1,1,L0)DO AMAX(I),
000141     24      SPAN,E,RHO,G,T,C,LBX,LPY,UBX,UBY,WMAX,SR                          )$
000141     25     REAL PROCEDURE MAS(X,Y)$REAL X,Y$ BEGIN
BLOCK 2     LEVEL 2                                                                         B2
000150     26     MAS= (864.0/32.2)RHO*X*Y(SPAN/V0)(1.0-SR)$                                E2     C
000211     27     END MASS
  END BLOCK 2
000212     28     REAL PROCEDURE WGT(X,Y)$REAL X,Y$BEGIN
BLOCK 3     LEVEL 2                                                                         B3     E3     C
000221     29     WGT=(32.2)*(V0)*MAS(X,Y)$END WGT$
  END BLOCK 3
000237     30         REAL PROCEDURE IALPHA(X,Y)$REAL X,Y $
BLOCK 4     LEVEL 2
000246     31      BEGIN                                                                    B4
000246     32     REAL X0,Y0,TAU0 $
000246     33     X0=X-2*DT*SQRT((X/Y)**2+1) $
000272     34     Y0=Y-2*DT*SQRT(1.0+1.0/((X/Y)**2))$
000321     35     TAU0=X0/Y0$
000324     36     IALPHA=1.118*RHO(SPAN/V0)(X(Y**3)((X/Y)**2+1)-X0(Y0**3)((X0/Y0)**2+1))$
000410     37     WRITE('X0,Y0,TAU0',X0,Y0,TAU0)$                                           E4     C
000416     38       END IALPHA $
  END BLOCK 4
000417     39     REAL PROCEDURE BSC(X,Y)$REAL X,Y$BEGIN
```

```
BLOCK 5     LEVEL 2
000426      ESC=(((8.0/E)*(SPAN/V0)**3))/((X**3)Y-((X-2*DT*SQRT(TAU*TAU+1.0))**3)     B5
000510        (Y-2*DT*SQRT(1.0+1.0/(TAU*TAU))))$
000547      END BSC$                                                                  E5   C
  END BLOCK 5
000550      REAL PROCEDURE TSC(X,Y)$REAL X,Y$BEGIN
BLOCK 6     LEVEL 2
000557      TSC=(2.0*SQRT(TAU*TAU+1.0)SPAN)/(G*X*X*Y*DT*V0)$                          B6
000632      END TSC $                                                                 E6   C
  END BLOCK 6
000633      REAL PROCEDURE CHB(X,Y,Z)$REAL X,Y,Z $  BEGIN
BLOCK 7     LEVEL 2
000643        CHB=SQRT(1.0/(MAS(X,Y)*BSC(X,Y)*Z))$                                    B7
000666      END CHB $                                                                 E7   C
  END BLOCK 7
000667      REAL PROCEDURE CHT(X,Y,Z)$REAL X,Y,Z$     BEGIN
BLOCK 8     LEVEL 2
000677        CHT=SQRT(1.0/(IALPHA(X,Y)*TSC(X,Y)*Z))$                                 B8
000722      END CHT$                                                                  E8   C
  END BLOCK 8
000723               PROCEDURE ANAL1(X,Y)$REAL X,Y$
BLOCK 9     LEVEL 2
000732               BEGIN
000732        INTEGER I,J,K,S,P,G,R,N,V,L   $                                         B9
000732         REAL LB,LT,FP,KH,KA,A,B,D,F,LAMBDA,LAMBDA1,LAMBDA2,MASS,RCG2,    M,
000732         XCG,REM,AA,AB,CC,MF1,MF2,RI1,RI2,RR1,RR2,RF1,RF2,DFS1,DFS2,ONE1,ONE2,
000732      TWO1,TWO2,THREE                                   $
000732        REAL ARRAY E1(1..11,1..10),H(1..10,1..10),X1(1..5),X2(1..5),
000762        IC(1..10,1..10), V1(1..5),U(1..11),MU(1..5)             $
001010         WRITE('    ENTER ANAL1  '  )$
001013       N=N0$V=V0$L=L0$
001027      TAU=X/Y $
001040      DT=0.5*Y((SQRT(TAU*TAU+1.0)-SQRT((TAU*TAU+1.0)-TAU(1.0-SR)SQRT(1.0+1.0/
001100      (TAU*TAU)+2.0)))/SQRT(TAU*TAU+1.0/(TAU*TAU)+2.0))$
001163      WRITE(' DT IS ')$WRITE(X,Y,DT)$
001173      FOR I=(1,1,V)DO BEGIN FOR J=(1,1,V)DO BEGIN                               B10
001245        IF I GTR J THEN H(I,J)= (I-J)**3+3(J-0.5)((I-0.5)**2)                   B11
001301        -((I-0.5)**3)$
001322      H(I,J)=3(I-0.5)(I-0.5)**2-(I-0.5)**3 END END $                            E11  E10
001357       FOR I=(2,1,V)DO BEGIN FOR J=(1,1,I-1)DO                                  B12
001434       H(I,J)=H(J,I) END$                                                       E12
001451       P=1$Q=1$
001455      RATS.. IF Q EQL 2 THEN BEGIN P=1$                                         B13
001466      FOR I=(1,1,V)DO BEGIN FOR J=(1,1,V)DO                                     B14
001540      H(I,J)=(I-0.5)END$                                                        E14
001553       FOR I=(2,1,V)DO BEGIN FOR J=(1,1,I-1)DO                                  B15
001630      H(I,J)=H(J,I)END END$                                                     E15  E13
001645      START.. FOR I=(1,1,V)DO X1(I)=X0(I)$
001706      REPIT.. FOR I=(1,1,V)DO X2(I)=0.0$
001741       FOR I=(1,1,V)DO BEGIN FOR J=(1,1,V)DO                                    B16
002013      X2(I)=X2(I)+(H(I,J))*(X1(J))END$                                          E16
002041      J=1$
002043      AGAIN..K=0$FOR I=(1,1,V)DO
002072      BEGIN IF ABS(X2(I)) GTR ABS(X2(J)) THEN                                   B17
```

```
002115   K=1$ENDS                                                         E17
002120     IF K EQL 1 THEN GO TO CYCLE $
002131    LAMBDA=X2(J)$
002136    FOR I=(1,1,V)DO X2(I)=(X2(I))/LAMBDA$
002176    GO TO LIKE$
002200     CYCLE.. J=J+1$ GO TO AGAIN $
002205   LIKE.. K=0$FOR I=(1,1,V)DO BEGIN
002234   IF ABS((X2(I)-X1(I))/X2(I))GTR 0.000001 THEN                     B18
002264   K=1$ ENDS                                                        E18
002267   IF K EQL 1 THEN BEGIN FOR I=(1,1,V)DO X1(I)=X2(I)$               B19
002335    GO TO REPIT ENDS                                                E19
002337   IF Q EQL 1 THEN
002346   BEGIN
002346   LB=CHB(X,Y,LAMBDA)$LAMBDA=LB$ ENDS                               B20   E20
002356   IF Q EQL 2 THEN BEGIN
002365   LT=CHT(X,Y,LAMBDA)       $LAMBDA=LT ENDS                         B21   E21
002375   IF Q EQL 1 THEN BEGIN FOR I=(1,1,V)DO                            B22
002431   NRMBD(I)=X2(I) ENDS                                              E22
002445   IF Q EQL 2 THEN BEGIN FOR I=(1,1,V)DO                            B23
002501    NRMTR(I)=X2(I) ENDS                                             E23
002515     Q=Q+1$ IF Q GTR 2 THEN GO TO STIFF $ GO TO RATS $
002534   STIFF..
002534   WRITE('LB,LT',LB,LT)$
002541     WRITE('IALPHA',IALPHA(X,Y))$
002545   MASS=MAS(X,Y)$
002552   FOR I=(1,1,V)DO BEGIN FOR J=(1,1,V)DO BEGIN IF I GTR J THEN      B24   B25
002654     IC(I,J)=((I-J)**3)+3(J-0.5)((I-0.5)**2)-((I-0.5)**3)$
002701     IC(I,J)=3(J-0.5)(I-0.5)**2-(I-0.5)**3 END END $                E25   E24
002736   FOR I=(2,1,V)DO BEGIN FOR J=(1,1,I-1)DO                          B26
003013   IC(I,J)=IC(J,I)ENDS                                              E26
003030     FOR I=(1,1,V)DO BEGIN
003055   FOR J=(1,1,V)DO IC(I+V,J+V)=(I-0.5)ENDS                          B27   E27
003122   FOR I=(2,1,V)DO BEGIN
003147   FOR J=(1,1,I-1)DO IC(I+V,J+V)=IC(J+V,I+V)ENDS                    B28   E28
003226   FOR I=(1,1,V)DO BEGIN FOR J=(1,1,V)DO                            B29
003300   IC(I,J)=ESC(X,Y)*IC(I,J)ENDS                                     E29
003323   FOR I=(V+1,1,2*V)DO BEGIN FOR J=(V+1,1,2*V)DO                    B30
003405   IC(I,J)=TSC(X,Y)*IC(I,J)ENDS                                     E30
003430   FOR I=(1,1,2*V)DO BEGIN FOR J=(1,1,2*V)DO B1(I,J)=IC(I,J)ENDS    B31   E31
003525   FOR I=(1,1,2*V)DO BEGIN
003555   IF B1(1,1) EQL 0.0 THEN GO TO HELPS                              B32
003573   FOR J=(1,1,2*V)DO U(J)=B1(1,J)/B1(1,1)$
003644   U(2*V+1)=(1.0)/B1(1,1)$FOR K=(1,1,2*V-1)DO
003712   BEGIN FOR J=(1,1,2*V-1)DO B1(K,J)=B1(K+1,J+1)-B1(K+1,1)*U(1+J)$  B33
004003   B1(K,2*V)=-B1(K+1,1)*U(2*V+1)ENDS                                E33
004032   FOR J=(1,1,2*V)DO B1(2*V,J)=U(1+J)ENDS                           E32
004100   RCG2=(4*IALPHA(X,Y))/(MAS(X,Y)*(Y**2))$
004123   WRITE('RCG2',RCG2)$
004127   FOR I=(1,1,5)DO BEGIN FOR J=(1,1,5)DO                            B34
004201   B1(I,J)=B1(I,J)/(MASS*LB**2) ENDS                                E34
004223   FOR I=(6,1,10)DO BEGIN FOR  J=(6,1,10)DO                         B35
004275   B1(I,J)=4*B1(I,J)/((LT**2)MASS(Y**2)RCG2) ENDS                   E35
004330   FOR I=(1,1,5)DO V1(I)=0.0$
004363   FOR I=(1,1,5)DO BEGIN FOR J=(1,1,5)DO                            B36
```

```
004435   V1(I)=V1(I)+NRMBD(J)*B1(J,I) ENDS                                  E36
004465   KH=0.0$
004467   FOR I=(1,1,5)DO KH=KH+V1(I)*NRMBD(I)$
004532   FOR I=(1,1,5)DO V1(I)=0.0$
004565   FOR I=(1,1,5)DO BEGIN FOR J=(1,1,5)DO                              B37
004637   V1(I)=V1(I)+NRMTR(J)*B1(J+5,I+5) ENDS                              E37
004674   KA=0.0$
004676   FOR I=(1,1,5)DO KA=KA+     V1(I)*NRMTR(I)$
004741   A=0.0$
004743   FOR I=(1,1,5)DO A=A+NRMBD(I)*NRMBD(I)$
005010   B=0.0$FOR I=(1,1,5)DO B=B+          NRMBD(I)*NRMTR(I)$
005057   D=0.0$FOR I=(1,1,5)DO D=D+NRMTR(I)*NRMTR(I)$
005126   FR=(LE/LT)**2$ GO TO FLUT$
005134   HELP..
005134     FLUT..     TAU=X/Y  $
005145     FOR I=(1,1,L)DO MU(I)=((664.0*RHO*X)/((32.2)Y*RHOA(I)))(1.0-SR)$
005237   FOR I=(1,1,L)DO BEGIN
005264   COMMENT INITIAL GUESS MACH MUMBERS M=3.0$                          B38
005266   RCG2=0.25$XCG=0.2$G=0$REM=5.0$
005276   STAR..
005276   F=A*D-B*B$
005307   AA=MU(I)*RCG2*KH*KA*FR*F*TAU-(1.0/3.0)MU(I)(D*D)(KH*KH)
005343   (FR*FR)TAU-A*D*MU(I)*RCG2*KH*KA*FR*TAU$
005402   BB=-D*F*MU(I)*RCG2*KH*FR*TAU-A*F*KA*MU(I )*RCG2*TAU
005433   +(2.0/3.0)MU(I)(D*D)A*KH*FR*TAU-(5.0/27.0)A(D*D)KH*FR(1.0/(M*M))
005504   +(5.0/9.0)MU(I)*XCG*KH*FR*B*B*D*(1.0/M)+
005535   (1.0/5.0)B*B*D*KH*FR*TAU*TAU
005550   +(A*A)D*MU(I)*RCG2*KA*TAU+A(D*D)MU(I)*RCG2*KH*FR*TAU
005611   -(5.0/9.0)(A*A)D*RCG2*KA(1.0/(M*M))+
005641   ((5.0/3.0)A*B*B*MU(I)*RCG2*XCG*KA)/M
005660   +(3.0/5.0)A*B*B*RCG2*KA(TAU*TAU)+
005704   (6.0/5.0)B*B*D*MU(I)*XCG*KH*FR*M(TAU*TAU)$
005737   CC=A*D*F*MU(I)*RCG2*TAU-B*F*MU(I)*(XCG*XCG)TAU *B
005774   -(1.0/3.0)(A*A )(D*D)MU(I)*TAU
006021   +((5.0/27.0)(A*A)(D*D))/(M*M)-((5.0/9.0)A*B*B*D*MU(I)*XCG)/M
006066   -(1.0/5.0)A*B*B*D(TAU**2)-(A*A)(D*D)MU(I)*RCG2*TAU
006130   +((5.0/9.0)(A*A)(D*D)RCG2)/(M*M)-((5.0/3.0)A*B*B*D*MU(I)
006170   *RCG2*XCG)/M
006200   -(3.0/5.0)A*B*B*D*RCG2(TAU**2)-(6.0/5.0)A*B*B*D*MU(I)*XCG*M(TAU**2)
006251   +(((2.0/3.0)A*B*B*D*XCG*TAU)/M)-2.0(B*B)MU(I)*(XCG*XCG)TAU*B*B
006314   -(18.0/25.0)(B*B)(B*B)XCG*M(TAU**3)$
006351   LAMBDA1=-BB/(2*AA)+(SQRT((BB**2)-(4*AA*CC)))/(2*AA)$
006403   LAMBDA2=-BB/(2*AA)-(SQRT((BB**2)-(4*AA*CC)))/(2*AA)$
006437   ONE1=((5.0/9.0)MU(I)*D(KH*FR*LAMBDA1-A))/(TAU*F)$
006467   ONE2=((5.0/9.0)MU(I)*D(KH*FR*LAMBDA2-A))/(TAU*F)$
006517   TWO1=((5.0/3.0)A*MU(I)*RCG2(KA*LAMBDA1-D))/(TAU*F)$
006547   TWO2=((5.0/3.0)A*MU(I)*RCG2(KA*LAMBDA2-D))/(TAU*F)$
006577   THREE=-(2*B*B*MU(I)*XCG*M)/F$
006615   RF1=SQRT(1.0/(ONE1+TWO1+THREE))$
006625   RF2=SQRT(1.0/(ONE2+TWO2+THREE))$
006635   MF1=((Y/2.0)(LT))/(VAIR(I)*RF1*SQRT(LAMBDA1))$
006661   MF2=((Y/2.0)(LT))/(VAIR(I)*RF2*SQRT(LAMBDA2))$
006705   DFS1=1.0/(RF1*SQRT(LAMBDA1))$
006714   DFS2=1.0/(RF1*SQRT(LAMBDA2))$
```

```
006723   193   RI1=MU(I)*D(KH*FR*LAMBDA1-A)(1.0/3.0)+A*MU(I)*RCG2(KA*LAMBDA1-D)
006755   194   -((6.0/10.0)TAU*F)/(RF1*RF1)-(6.0/5.0)E*B*MU(I)*XCG*MF1*TAU$
007023   195   RI2=(1.0/3.0)MU(I)*D(KH*FR*LAMBDA2-A)+A*MU(I)*RCG2(KA*LAMBDA2-D)
007055   196   -((0.6)TAU*F)/(RF2*RF2)-(6.0/5.0)E*B*MU(I)*XCG*MF2*TAU$
007122   197   RR1=(MU(I)*MU(I))RCG2(KH*FR*LAMBDA1-A) *(KA*LAMBDA1-D)
007146   198   +(1.0/RF1*RF1 )(-(0.6)MU(I)*D*TAU(KH*FR*LAMBDA1-A)
007176   199   -(A*D)/(3(MF1*MF1))+(B*B*MU(I)*XCG)/MF1
007226   200   +(18.0/50.0)B*B(TAU*TAU))-B*B(MU(I)*MU(I))(XCG*XCG)$
007277   201   RR2=(MU(I)*MU(I))RCG2(KH*FR*LAMBDA2-A) *(KA*LAMBDA2-D)
007323   202   +(1.0/(RF2*RF2))(-(0.6)MU(I)*D*TAU(KH*FR*LAMBDA2-A)
007353   203   -(A*D)/(3(MF2*MF2))+(B*B*MU(I)*XCG)/MF2
007403   204   +(18.0/50.0)B*B(TAU*TAU))-B*B(MU(I)*MU(I))(XCG*XCG)$
007454   205   IF Q GTR 15 THEN BEGIN REM=REM+5.0$Q=0 END$                      B39   E39
007471   206      IF NOT(ABS(RR1) LSS REM AND ABS(RI1) LSS REM )
007514   207     THEN BEGIN G=G+1$M=MF1$GO TO STAR END$                          B40   E40
007526   208   MCH(I)=MF1         $             END$                             E38
007536   209   WRITE('MCH',MCH)$
007542   210            END ANALI$                                              E9    C
  END BLOCK 9
007543   211            REAL PROCEDURE PDRAG(X,Y,S)$REAL X,Y$INTEGER S$BEGIN
BLOCK 10    LEVEL 2
007553   212   REAL ARRAY H(1..10,1..10),D(1..10,1..10),B(1..10,1..10),          B41
007600   213   ALPO,ALP1,DEFL,ANGL,LIFT,MOMNT,DF,DP(1..10),U(1..11)                          $
007632   214   REAL ALPHO,DELTA,DELAN,S1,S2,SIGMA,DRAGP,LT2,MT2,X0,Y0          $
007637   215   INTEGER I,J,K,P,Q,R ,N,V           $
007637   216     WRITE(' ENTER PDRAG ')$
007642   217   N=N0$ V=V0 $
007652   218    ALPHO=((LF(S)+WGT(X,Y))/((2.8)SPAN*Y*PRES(S)*MACH(S)))(0.80) $
007731   219   OVER..FOR I=(1,1,N)DO BEGIN FOR J=(1,1,N)DO H(I,J)=0.0END$        B42   E42
010013   220   FOR I=(1,1,N)DO BEGIN FOR J=(1,1,N)DO H(I,J)=(I-0.5) END$         B43   E43
010100   221   FOR I=(2,1,N)DO BEGIN FOR J=(1,1,I-1)DO H(I,J)=H(J,I) END$        B44   E44
010172   222   FOR I=(1,1,N)DO ALPO(I)=ALPHO$
010225   223   FOR I=(1,1,N)DO BEGIN FOR J =(1,1,N)DO                            B45
010277   224   H(I,J)=1.66*PRES(S)*(MACH(S)**2)((SPAN/N)**2)(1.0/G)*(1.0/X)*(1.0/DT)*
010366   225   SQRT(TAU*TAU+1.0)*H(I,J) END$                                     E45
010413   226   FOR I=(1,1,N)DO ALP1(I)=0.0$
010446   227   FOR I=(1,1,N)DO BEGIN FOR K=(1,1,N)DO                             B46
010520   228   ALP1(I)=ALP1(I)+H(I,K)*ALPO(K) END$                               E46
010546   229   FOR I=(1,1,N)DO BEGIN FOR J=(1,1,N)DO D(I,J)=0.0 END $            B47   E47
010630   230   FOR I=(1,1,N)DO D(I,I)=1.0$
010664   231   FOR I=(1,1,N)DO BEGIN FOR J=(1,1,N)DO H(I,J)=D(I,J)-H(I,J) END$   B48   E48
010761   232   FOR I=(1,1,N)DO BEGIN FOR J=(1,1,N)DO B(I,J)=H(I,J) END$          B49   E49
011050   233   V=N/2 $
011056   234   COMMENT START INVERT $
011056   235   FOR I=(1,1,2*V)DO BEGIN
011106   236   IF B(1,1) EQL 0.0 THEN GO TO AID $                                B50
011124   237   FOR J=(1,1,2*V)DO U(J)=B(1,J)/B(1,1)$
011175   238   U(2*V+1)=(1.0)/B(1,1)$ FOR K=(1,1,2*V-1)DO
011243   239   BEGIN FOR J=(1,1,2*V-1)DO B(K,J)=B(K+1,J+1)-B(K+1,1)*U(1+J)$      B51
011334   240   B(K,2*V)=-B(K+1,1)*U(2*V+1) END$                                  E51
011363   241   FOR J=(1,1,2*V)DO B(2*V,J)=U(1+J) END$                            E50
011431   242   COMMENT END INVERT $
011431   243   FOR I=(1,1,N)DO ANGL(I)=0.0$
011464   244   FOR I=(1,1,N)DO BEGIN FOR K=(1,1,N)DO                             B52
```

```
011536   245      ANGL(I)=ANGL(I)+B(I,K)*ALP1(K) END $                                          E52
011564   246      FOR I=(1,1,N)DO ANGL(I)=ANGL(I)+ALPO(I)$
011630   247      ATA(S)=ANGL(1)$
011645   248               FOR I=(1,1,N)DO
011672   249               LIFT(I)=((2.8)PRES(S)*MACH(S)*Y*(SPAN/N))*ANGL(I) $
011747   250                LT2=0.0$
011751   251               FOR I=(1,1,N)DO LT2=LT2+LIFT(I)$
012005   252               DELTA=LT2-LF(S)$
012017   253                IF ABS(DELTA)LSS 100.0 THEN GO TO HOMES
012032   254       DELAN =(DELTA/((2.8)SPAN*Y*PRES(S)*MACH(S)))(0.20) $
012073   255                ALPHO=ABS(ALPHO-DELAN)$GO TO OVERS
012101   256      HOME.. FOR I=(1,1,N)DO
012126   257        MOMNT(I)=(3.36/4.0)*PRES(S)*X*Y*(MACH(S)**2)(SPAN/N)*ANGL(I)$
012213   258      WRITE('MOMENT',MOMNT)$
012217   259      FOR I=(1,1,N)DO BEGIN FOR J=(1,1,N)DO                                          B53
012271   260       H(I,J)=3(J-0.5)(I-0.5)**2-(I-0.5)**3 ENDS                                     E53
012326   261      FOR I=(2,1,N)DO BEGIN FOR J=(1,1,I-1)DO H(I,J)=H(J,I)ENDS                      B54    E 4
012420   262      FOR I=(1,1,N)DO BEGIN FOR J=(1,1,N)DO                                          B55
012472   263      H(I,J)=(6.0/(E((X**3)*Y-((X-2*DT*SQRT(TAU*TAU+1.0))**3)(Y-2*DT*SQRT(1+
012542   264      1.0/(TAU*TAU))))))((SPAN/N)**3)*H(I,J) END $                                  E55
012624   265      FOR I=(1,1,N)DO DEFL(I)=0.0$
012657   266      FOR I=(1,1,N)DO BEGIN FOR K=(1,1,N)DO                                          B56
012731   267      DEFL(I)=DEFL(I)+H(I,K)*LIFT(K) ENDS                                            E56
012757   268      WRITE('LIFT,DEFL',LIFT,DEFL)$
012764   269      DFL(S)=DEFL(10)+(Y/2)ANGL(10)$
013013   270       X0=X-2*DT*SQRT(TAU*TAU+1)$
013037   271       Y0=Y-2*DT*SQRT(1+1.0/(TAU*TAU))$
013066   272      LT2=MT2=0.0$
013071   273      FOR I=(1,1,N)DO BEGIN  MT2=MT2+MOMNT(I)$                                       B57
013124   274                                    LT2=LT2+(I-0.5)LIFT(I) END $                      E57
013141   275           S1=((12(SPAN/N)X)/(Y*X**3-Y0*X0**3))LT2 $
013204   276        S2=SQRT(S1**2+((1.0/(SR*X*Y*DT))MT2)**2) $
013235   277          SIGMA=SQRT(S1**2+3(S2**2))$
013246   278      STR(S)=SIGMA$
013257   279      FOR I=(1,1,N)DO
013304   280      DP(I)=((2.8)PRES(S)*MACH(S)*Y*(SPAN/N))*((ANGL(I)**2)+
013367   281      (X/Y)**2)$
013373   282      DRAGP=0.0$
013375   283      FOR I=(1,1,N)DO
013422   284      DRAGP=DRAGP+DP(I)$
013431   285      FDRAG=DRAGP$
013432   286        GO TO TERMS AID..
013435   287        TERM..
013435   288         END FDRAGS                                                                  E41    C
  END BLOCK 10
013436   289                     REAL PROCEDURE MAR(X,Y,W,S)$REAL X,Y,W$
BLOCK 11    LEVEL 2
013447   290                                                  INTEGER S$
013447   291                          BEGIN
013447   292                     MAR=(FDRAG(X,Y,S)+((SPAN/N0)W))VEL(S)*TIME(S)  $                B58
013522   293                     END MARS                                                         E58    C
  END BLOCK 11
013523   294      REAL PROCEDURE ENG(X,Y)$REAL X,Y $
BLOCK 12    LEVEL 2
```

```
013524   295              BEGIN
013524   296                 REAL SUM,KK,LL,HH,F1,F2,QQ,SS,XX                               $            B59
013524   297               INTEGER I,S,L                                        $
013524   298                          REAL PROCEDURE CON(S)$INTEGER S$
BLOCK 13    LEVEL 3
013532   299                          BEGIN CON=(0.259*(MACH(S)**2)PRES(S))/(1.0                          B60
013556   300                         +0.13(MACH(S)**2))$                                                  E60   C
01357б   301                           END CON$
  END BLOCK 13
013577   302                          REAL PROCEDURE STA(S)$INTEGER S$
BLOCK 14    LEVEL 3
013605   303                          BEGIN STA=(1.2260 )MACH(S)*PRES(S)(TEMP(S)*(1.0                     B61
013640   304                           +0.13(MACH(S)**2))+198.7)$                                         E61   C
013660   305                           END STA$
  END BLOCK 14
013661   306                             REAL PROCEDURE NT(S)$INTEGER S$
BLOCK 15    LEVEL 3
013667   307                                BEGIN NT=(TEMP(S)*TEMP(S))(1.0                                B62
013710   308                                   +0.13(MACH(S )**2))**(2.5)$                                E62   C
013730   309                                    END NT $
  END BLOCK 15
013731   310                                   REAL PROCEDURE ANOT(S)$INTEGER S$
BLOCK 16    LEVEL 3
013737   311                                   BEGIN ANOT=2*COS(ATA(S))$                                  B63
013752   312                                END ANOT$                                                     E63   C
  END BLOCK 16
013753   313                                   REAL PROCEDURE SKNFN(Z,S)$REAL Z$
BLOCK 17    LEVEL 3
013762   314        INTEGER S $
013762   315                         BEGIN                                                                B64
013762   316         SKNFN=(CON(S)*(0.43429*LN(STA(S)*Z/NT(S)))**(-2.584))ANOT(S)$                        E64   C
014022   317                    END SKNFN  $
  END BLOCK 17
014023   318      L=L0$
014027   319      SUM=0.0$ FOR S=(1,1,L)DO BEGIN                                                          B65
014056   320      KK=LL=SKNFN(0.01Y,S)+SKNFN(Y,S)$ HH=Y-0.01Y$F1=F2=0.0$
014107   321      GO TO ITER       $IF ABS((F2-F1)/F2)GTR EPN THEN                                         B66
014127   322      BEGIN ITER.. QQ=HH/2.0$ SS=0.0$ F1=F2$
014136   323      FOR XX=(0.01Y+QQ,HH,Y)DO SS=SS+SKNFN(XX,S)$
014202   324      KK=LL+4*SS$LL=LL+2*SS$HH=QQ$
014214   325      WRITE('F1,F2',F1,F2)$                                                                   E66
014221   326      F2=(10.0)KK*QQ/(3.0) END$                                                               E65
014226   327      SUM=SUM+MAR(X,Y,F2,S) END$
014237   328      ENG=SUM $                                                                               E59   C
014241   329          END ENG$
  END BLOCK 12
014242   330      PROCEDURE NORM(X,Y)$REAL X,Y$
BLOCK 18    LEVEL 2                                                                                       B67
014251   331      BEGIN REAL ARRAY A(1..8)$
014256   332      INTEGER I,K,M$
014256   333      REAL DRX,DRY,Z      $
014256   334      A(1)=X/10.0 $ A(2)=Y/10.0$
014276   335      DRX=(ENG(X+A(1),Y)-ENG(Y-A(1),Y))/(2*A(1))$
014324   336      DRY=(ENG(X,Y+A(2))-ENG(X,Y-A(2)))/(2*A(2))$
```

```
014352   337    XCOMP=-DFX/(DRX*DRX+DRY*DRY)$
014370   338    YCOMP=-DFY/(DRX*DRX+DRY*DRY)$
014406   339    EX=ABS(DFX/SQRT(DRX*DRX+DRY*DRY))$
014424   340    EY=ABS(DFY/SQRT(DRX*DRX+DRY*DRY))$
014442   341    END NORM$                                                       E67   C
  END BLOCK 18
014443   342    REAL PROCEDURE CONSTRAINT(X,Y)$REAL X,Y$
BLOCK 19    LEVEL 2
014452   343    BEGIN
014452   344            REAL ARRAY FC(1..LN)$                                   B68
014462   345            INTEGER I,J,K,L  $
014462   346     WRITE('ENTER CONSTRAINT ')$WRITE(X,Y)$
014471   347    L=LN$
014475   348            FOR I=(1,1,L)DO
014522   349    BEGIN
014522   350            FC(I)=MAX(DFL(I)/DMAX(I),STR(I)/SMAX(I),                B69
014526   351    ATA(I)/AMAX(I),MACH(I)/MCH(I),WGT(X,Y)/WMAX) $
014536   352    END$                                                            E69
014537   353     WRITE(LFL,DMAX,STR,SMAX,ATA,AMAX,MACH,MCH,WGT(X,Y),WMAX)$
014553   354            J=1$
014555   355             AGAIN.. K=0$ FOR I=(1,1,L)DO
014604   356            BEGIN IF ABS(FC(I)) GTR ABS(FC(J)) THEN                 B70
014627   357    K=1   END $                                                     E70
014632   358            IF K EQL 1 THEN BEGIN
014641   359            J=J+1 $ GO TO AGAIN  END$                               B71   E71
014646   360            CONSTRAINT = FC(J) $
014653   361    WRITE(FC)$
014656   362         END CONSTRAINT $                                           E68   C
  END BLOCK 19
014657   363    PROCEDURE TEST(X,Y,ON,FREE,NOT)$
BLOCK 20    LEVEL 2
014671   364    REAL X,Y$ LABEL ON,FREE,NOT$
014671   365    BEGIN REAL Z$                                                   B72
014671   366     WRITE(' ENTER TEST')$WRITE(X,Y)$
014700   367       WRITE( DFL,STR,ATA,MCH,MACH,MERIT                     )$
014700   368    IF((X LSS LBX)OR(Y LSS LBY)OR(X GTR UBX)OR (Y GTR UBY)) THEN
015002   369    GO TO NOT$
015006   370    IF((X EQL LBX)OR(Y EQL LBY)OR(X EQL UBX)OR(Y EQL UBY))THEN BEGIN
015074   371    Z=CONSTRAINT(X,Y)$                                              B73
015101   372    IF(Z GTR 1.0)THEN GO TO NOT$
015116   373    IF(Z GTR 1.0-EPS)THEN GO TO ON$
015140   374    GO TO FREE END $                                                E73
015144   375    Z=CONSTRAINT(X,Y)$
015151   376    IF(Z GTR 1.0)THEN GO TO NOT$
015166   377    IF(Z GTR 1.0-EPS)THEN GO TO ON$
015210   378    GO TO FREE$
015214   379    END TEST $                                                      E72   C
  END BLOCK 20
015215   380    PROCEDURE CHECK(X,Y,XCOMP,YCOMP,XS,YS)$
BLOCK 21    LEVEL 2
015230   381    REAL X,Y,XCOMP,YCOMP$
015230   382    INTEGER XS,YS$
015230   383    BEGIN IF PER EQL 0.0 THEN PER=1.0$                              B74
015246   384      WRITE(    '  CHECK ENTERED ')$
```

```
015251   385     WRITE(X,Y)$
015255   386     IF((X LSS LBX)OR(Y LSS LBY)OR(X GTR UBX)OR
015327   387     (Y GTR UBY)) THEN
015347   388     BEGIN IF NOT((X GTR LBX)AND(Y GTR LBY)AND(X LSS UBX)AND(Y LSS UBY))THEN            B75
015443   389     BEGIN X=X+XS*0.33*EPS*PER*MERIT*XCOMP$                                             B76
015475   390           Y=Y+YS*0.33*EPS*PER*MERIT*YCOMP$
015527   391      WRITE('CHECK CYCLE')$ WRITE(X,Y)$
015536   392     END END $                                                                          E76    E75
015536   393     END CHECK $                                                                        E74    C
  END BLOCK 21
015537   394     PROCEDURE BOUND(X,Y,XCOMP,YCOMP)$
BLOCK 22    LEVEL 2
015550   395     REAL X,Y,XCOMP,YCOMP$
015550   396     BEGIN
015550   397       WRITE('BOUND ENTERED')$                                                          B77
015553   398     WRITE(X,Y)$
015557   399     MERIT=ENG(X,Y)$
015566   400     SEE..IF((MERIT-MERON)GTR 0.0)THEN
015605   401     BEGIN X=X+(1.0)*MERIT-MERON)XCOMP$                                                 B78
015630   402           Y=Y+(1.0)*MERIT-MERON)YCOMP$
015653   403           MERIT=ENG(X,Y)$
015662   404       WRITE('BOUND CYCLE ')$ WRITE(X,Y)$
015671   405            GO TO SEE END $                                                             E78
015673   406     END BOUND $                                                                        E77    C
  END BLOCK 22
015674   407      EPS=0.01        $
015676   408       EPN=0.1$
015700   409     READ(RD)$
015703   410     WRITE(RD)$
015706   411     COMMENT SYNTHESIS BEGINS HERE $
015706   412     ANAL1(T,C)$
015712   413     MERIT=ENG(T,C)$
015717   414     TEST(T,C,ON,FREE,NO)$
015726   415     FREE.. NORM(T,C)$
015732   416     WRITE('PROGRAM AT   FREE      ')$
015735   417           T=T+0.5*MERIT*XCOMP$
015742   418           C=C+0.5*MERIT*YCOMP $
015747   419           CHECK(T,C,XCOMP,YCOMP,-1,-1)$
015757   420           ANAL1(T,C)$
015763   421            MERIT=ENG(T,C)$
015770   422     TEST(T,C,ON,FREE,NO)$
015777   423     NO..   NORM(T,C)$
016003   424     WRITE('PROGRAM AT   NO        ')$
016006   425           PER=0.5$
016010   426           T=T-0.5*MERIT*XCOMP$
016017   427           C=C-0.5*MERIT*YCOMP$
016026   428           CHECK(T,C,XCOMP,YCOMP,1,1)$
016036   429           ANAL1(T,C)$
016042   430           MERIT=ENG(T,C)$
016047   431           TEST(T,C,ON,F1,N1)$
016056   432     N1..    PER=0.5*PER$
016061   433     WRITE('PROGRAM AT   N1        ')$
016064   434            IF PER LSS 0.0001 THEN PER=0.1$
016077   435            T=T-PER*MERIT*XCOMP$
```

```
016106   C=C-PER*MERIT*YCOMP$
016115   CHECK(T,C,XCOMP,YCOMP,1,1)$
016125   ANAL1(T,C)$
016131   MERIT=ENG(T,C)$
016136   TEST(T,C,ON,F1,N1)$
016145   F1..   PER=0.5*PER$
016150   WRITE('PROGRAM AT      F1     ')$
016153   IF PER LSS 0.0001 THEN PER=0.1$
016166   T=T+PER*MERIT*XCOMP$
016173   C=C+PER*MERIT*YCOMP$
016200   CHECK(T,C,XCOMP,YCOMP,-1,-1)$
016210   ANAL1(T,C)$
016214   MERIT=ENG(T,C)$
016221   TEST(T,C,ON,F1,N1)$
016230   NO1..  NORM(T,C)$
016234   WRITE('PROGRAM AT   NO1       ')$
016237   T=T1-0.5*MERIT*XCOMP$
016246   C=C1-0.5*MERIT*YCOMP$
016255   CHECK(T,C,XCOMP,YCOMP,1,1)$
016265   ANAL1(T,C)$
016271   MERIT=ENG(T,C)$
016276   TEST(T,C,ON,FREE,NO)$
016305   ON..   IF(CONSTRAINT(T,C) GTR 1.0) THEN
016321   BEGIN                                              B79
016321   PER=0.5$
016323   NORM(T,C)$
016327   GO TO N1$                                          E79
016331   END$
016331   WRITE('PROGRAM AT   ON         ')$
016334   WRITE('BOUND POINT NE TAN MOVE')$
016337   WRITE(T,C)$
016343   NORM(T,C)$
016347   COMMENT TAN MOVE NE $
016347   TCOMP=XCOMP$CCOMP=YCOMP$ MERON=MERIT$
016355   T1=T+MERON*CCOMP$
016361   C1=C-MERON*TCOMP$
016367   WRITE('TAN MOVE COMPS ARE')$ WRITE(T1,C1)$
016376   CHECK(T1,C1,YCOMP,XCOMP,-1,+1)$
016406   BOUND(T1,C1,TCOMP,CCOMP) $
016414   ANAL1(T1,C1)$
016420   MERIT=ENG(T1,C1)$
016425   TEST(T1,C1,ON1,F2,NOT2)$
016434   F2..   T=T1$C=C1$PER=0.3$GO TO F1$
016444   ON1..  T1=T1+MERON*CCOMP$C1=C1-MERON*TCOMP$
016456   WRITE('PROGRAM AT   ON1       ')$
016461   WRITE(' BOUND POINT,FIR TAN MOVE DOUB ')$
016464   CHECK(T1,C1,YCOMP,XCOMP,-1,+1)$
016474   BOUND(T1,C1,TCOMP,CCOMP)$
016502   ANAL1(T1,C1)$
016506   MERIT=ENG(T1,C1)$
016513   TEST(T1,C1,ON1,F2,NOT2)$
016522   NOT1..  COMMENT SEC DIR TAN MOVE $
016522   WRITE('PROGRAM AT   NOT1      ')$
016525   WRITE(' BOUND POINT SW TAN MOVE ')$
```

```
016530   490            T1=T-MERON*CCOMP$C1=C+MERON*TCOMP$
016542   491            CHECK(T1,C1,YCOMP,XCOMP,+1,-1)$
016552   492            BOUND(T1,C1,TCOMP,CCOMP)$
016560   493            ANAL1(T1,C1)$
016564   494            MERIT=ENG(T1,C1)$
016571   495            TEST(T1,C1,ON2,F2,NOT3)$
016600   496      ON2..   T1=T1-MERON*CCOMP$C1=C1+MERON*TCOMP$
016612   497      WRITE('PROGRAM AT   ON2       ')$
016615   498            WRITE('BOUND POINT,SEC DIR TAN MOVE DOUB ')$
016620   499            CHECK(T1,C1,YCOMP,XCOMP,+1,-1)$
016630   500            BOUND(T1,C1,TCOMP,CCOMP)$
016636   501            ANAL1(T1,C1)$
016642   502            MERIT=ENG(T1,C1)$
016647   503            TEST(T1,C1,ON2,F2,NOT3)$
016656   504      NOT2..  COMMENT DEDUCE TAN MOVE FIR DIR $
016656   505      WRITE('PROGRAM AT    NOT2      ')$
016661   506            T1=T+0.5*MERON*CCOMP$C1=C-0.5*MERON*TCOMP$
016675   507            CHECK(T1,C1,YCOMP,XCOMP,-1,+1)$
016705   508            BOUND(T1,C1,TCOMP,CCOMP)$
016713   509            ANAL1(T1,C1)$
016717   510            MERIT=ENG(T1,C1)$
016724   511            TEST(T1,C1,NOT1,F2,NOT1)$
016733   512      NOT3.. COMMENT REDUCE TAN MOVE SEC DIR $
016733   513      WRITE('PROGRAM AT    NOT3      ')$
016736   514            T1=T-0.5*MERON*CCOMP$
016745   515            C1=C+0.5*MERON*TCOMP$
016752   516            CHECK(T1,C1,YCOMP,XCOMP,+1,-1)$
016762   517            BOUND(T1,C1,TCOMP,CCOMP)$
016770   518            ANAL1(T1,C1)$
016774   519            MERIT=ENG(T1,C1)$
017001   520            TEST(T1,C1,DONE,F2,DONE)$
017010   521         COMMENT END OF SYNTHESIS$
017010   522      DONE.. ANAL1(T,C)$
017014   523          MERIT=ENG(T,C)$
017021   524      WRITE('PROGRAM AT   DONE      ')$
017024   525         WRITE(' THE SYNTHESIS RESULTS ARE THE FOLLOWING ')$
017027   526          WRITE(T,C,MERIT,DFL,MCH,ATA,STR,CONSTRAINT(T,C),WGT(T,C)    )$
017042   527                END$                                                          E1
  END BLOCK 1
017043   528        FINISH     $                                                         F0
COMPILATION COMPLETE
```